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Computation of Impacts on Elastic Solids by Methods of Bicharacteristics

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Shock waves, explosions, impacts or cavitation bubble collapses may generate stress waves in solids causing cracks or unexpected dammage due to focussing, physical nonlinearity or interaction with existing cracks. There is a growing interest in wave propagation, which poses many novel problems to experimentalists and theorists.

Wave propagation ist mostly governed by hyperbolic PDEs. Numerical methods applicable to strong waves in solids, which are as powerfull as the FEM e.g. applied to static or quasi-static problems, have not yet been evolved for multi-dimensional problems. All state of the art methods for hyperbolic problems are based on the solution of the local wave propagation. They differ in the construction of the fundamental solution and thus in the treatment of singularities connected with any sharp wave front such as acceleration waves and shocks. For application to solids, the methods of bicharacteristics seem to be especially well suited, since fundamental solutions can be constructed in a straight forward way for various constitutive equations |1|.

The balance of momentum (1) and the material time derivative of the constitutive equation of an elastic solid (2) form a hyperbolic first order system for positive definite fourth order stiffness $\mathbf{A}$

$$\mathbf{l} = \mathbf{o} \;, \; \mathbf{l} := \overline{\rho}\,\frac{\partial \mathbf{v}}{\partial \tau} - \overline{\mathrm{div}}\,\overline{\boldsymbol{\sigma}} \;, \tag{1}$$

$$\mathbf{L} = \mathbf{0}, \;\; \mathbf{L} := \frac{\partial \overline{\boldsymbol{\sigma}}}{\partial \tau} - \mathbf{A} : \overline{\mathrm{grad}}\, \mathbf{v} \;. \tag{2a}$$

Multiple dots denote multiple transvection and dashes indicate the formulation of a quantity in the reference configuration. τ is the time, ρ is the mass density and $\mathbf{v}$ is the particle velocity. In linearized theory with small deformations and small displacements the material differential operators are replaced by the spatial ones. Furthermore the first PIOLA– KIRCHHOFF stresses $\overline{\boldsymbol{\sigma}}$ merge into the symmetric CAUCHY stresses $\boldsymbol{\sigma}$ and only the symmetric part of grad $\mathbf{v}$ remains. Then for $\mathbf{A}$ a given function in space

$$\mathbf{L} := \frac{\partial \boldsymbol{\sigma}}{\partial \tau} - \frac{1}{2}\,\mathbf{A} : (\mathrm{grad}\,\mathbf{v} + (\mathrm{grad}\,\mathbf{v})^{\mathrm{T}}) \;, \tag{2b}$$

holds for an anisotropic linear elastic solid and may be further specified for an isotropic one, yielding

$$\mathbf{L} := \frac{\partial \boldsymbol{\sigma}}{\partial \tau} - \lambda\, \mathbf{E}\, \mathrm{div}\, \mathbf{v} + \mu\, (\mathrm{grad}\, \mathbf{v} + (\mathrm{grad}\, \mathbf{v})^T)\ , \tag{2c}$$

where λ, μ are the LAME constants.

For nonlinear solids generally $\mathbf{A}$ cannot be specified as a function of $\bar{\boldsymbol{\sigma}}$. Therefore the deformation gradient is usually chosen as unknown function instead of $\bar{\boldsymbol{\sigma}}$ |2,3|.

For wave problems in plates disturbances emanating from a point in space and time propagate along MONGE-cones, which are enveloped by all plane waves or singular surfaces passing through the point and are generated by the so-called bicharacteristics $\bar{\mathbf{m}}^*$. The characteristic condition is an equation for the normals $\bar{\mathbf{n}}^*_\varepsilon = -\nu_\varepsilon / c\, \bar{\mathbf{g}}_0 + \bar{\mathbf{n}}$, ($\varepsilon$=0,L,T), of singular surfaces. Its solutions are $\nu_0 = 0$ and the speeds ν_L of quasi-longitudinal and ν_T of quasi-transversal waves.

Here c is an arbitrary constant speed, $\bar{\mathbf{g}}_0$ the time-like base vector, $\bar{\mathbf{n}}$ the space-like normal, $|\bar{\mathbf{n}}| = 1$. $\nu_L(\bar{\mathbf{n}})$, $\nu_T(\bar{\mathbf{n}})$ are calculated from the eigenvalues of the local acoustic tensor $\mathbf{Q}(\bar{\mathbf{n}})$

$$\mathbf{Q}(\bar{\mathbf{n}}) := (\mathbf{g}_\lambda \circ \bar{\mathbf{n}} : \mathbf{A} : \mathbf{g}_\mu \circ \bar{\mathbf{n}})\, \mathbf{g}^\lambda \circ \mathbf{g}^\mu\ . \tag{3}$$

$\mathbf{Q}$ is symmetric for hyperelastic materials and has eigenvectors $\mathbf{q}_\eta$ (η= L,T). A vector basis in a singular surface is formed by the space-like tangent $\bar{\mathbf{t}}$ ($|\bar{\mathbf{t}}| = 1$) of the MONGE cones and its generators $\bar{\mathbf{m}}^*_\varepsilon$ or any near characteristic $\bar{\mathbf{s}}^*_\varepsilon$. The PDEs (1), (2) admit discontinuous derivatives which are undetermined in direction of $\bar{\mathbf{n}}^*_\varepsilon$. All derivatives within singular surfaces are continuous. The undetermined derivatives can be eliminated by linear combinations |3|, resulting in the so-called compability equations

$$\mathbf{o} = \mathbf{A}^{-1} \vdots (\bar{\mathbf{t}} \circ \mathbf{L}) \qquad \text{for} \quad \nu_0\ , \tag{4}$$

$$0 = \mathbf{q}_\eta(\bar{\mathbf{n}})\, (\nu_\eta(\bar{\mathbf{n}})\, \mathbf{1} - \mathbf{L} \cdot \bar{\mathbf{n}}) \qquad \text{for} \quad \nu_\eta(\bar{\mathbf{n}})\ . \tag{5}$$

Only in homogeneous isotropic linear elastic plates point disturbances propagate along circular cones locally and globally in space and time. Anisotropy and inhomogeneity may produce complicated higher order cones and conoids |4|. Both effects may occur in nonlinear plates and depend on the local values of the solution |2,3|. It can be shown that the torsion of the conoids due to continuous inhomogeneities may be neglected in a second order difference scheme |3|. For anisoptropic wave propagation only a method of near characteristics is easy handled. But fortunately, choosing $\bar{\mathbf{s}}^*_\varepsilon := c\bar{\mathbf{g}}_0 + \nu_\varepsilon(\bar{\mathbf{n}})\, \bar{\mathbf{n}}$, it merges into a method of bicharacteristics on the axes of the symmetry of the MONGE cones, since then $\bar{\mathbf{s}}^*_\varepsilon = \bar{\mathbf{m}}^*_\varepsilon$.

All derivatives in (4), (5) are continuous and may be expanded into TAYLOR series and integrated numerically in a time interval $\Delta\tau$ along $\bar{\mathbf{s}}^*_\varepsilon$ or $\bar{\mathbf{m}}^*_\varepsilon$. In a second order scheme initial values may be calculated from a least squares approximation. At the solution point the derivatives in cross direction $\bar{\mathbf{t}}$ are additional unknowns, which can be eliminated by linear combinations of difference equations |3,5,6|.

Based on the theory above codes of different generality were written and applied successfully to various physical and technical problems: focussing of waves in

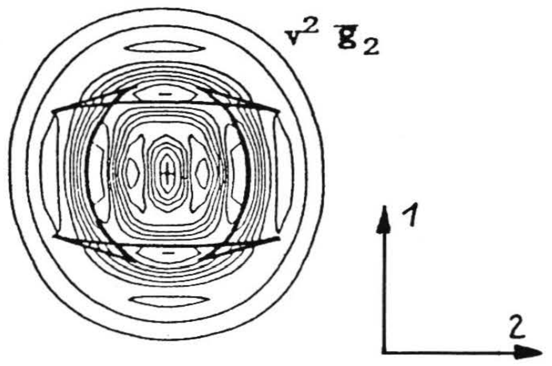

Fig. 1 Contour lines of velocities

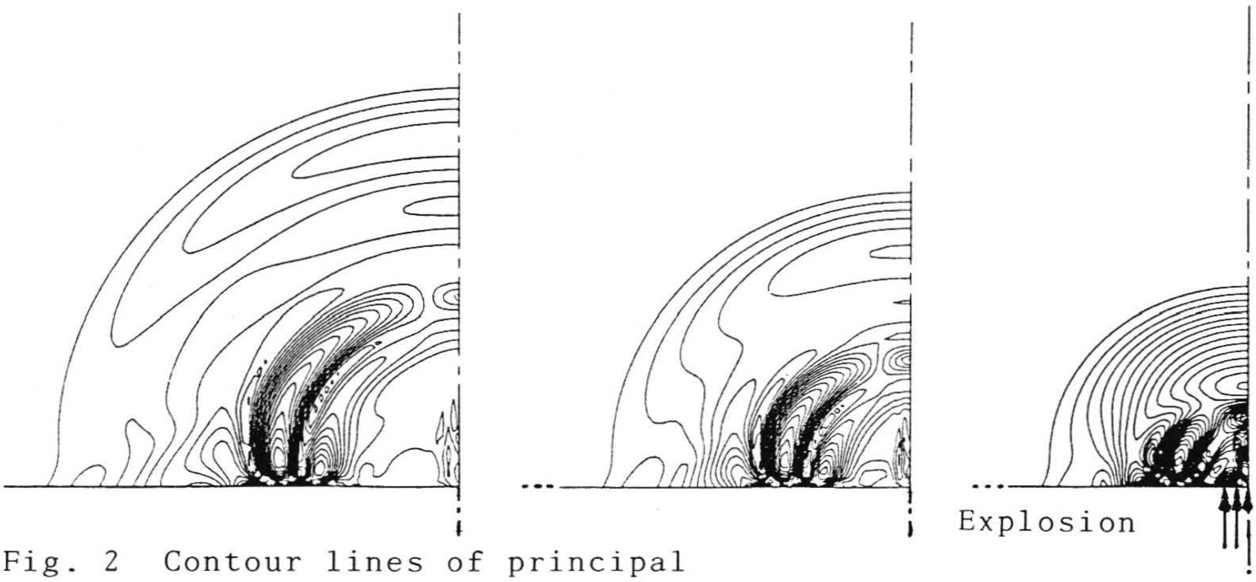

Fig. 2 Contour lines of principal shear stresses

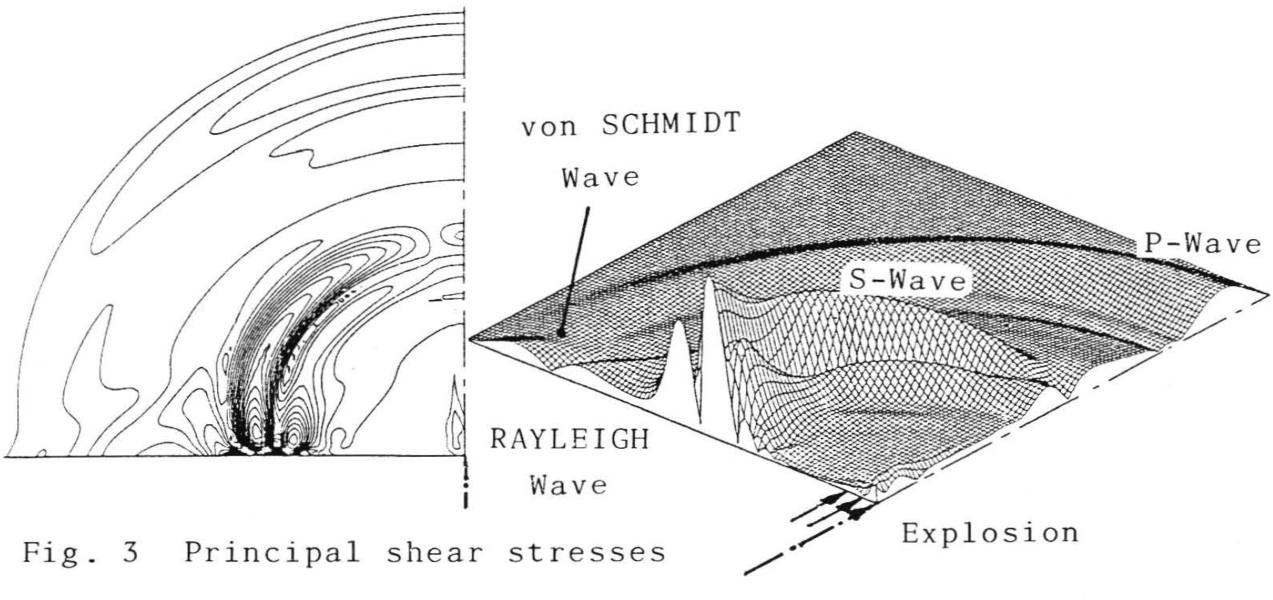

Fig. 3 Principal shear stresses

a linear semicircular plate |2,6|, VON SCHMIDT waves at a material interface |6|, nonlinear anisotropic wave propagation |2,3|, changes of wave profiles including the forming of shocks |3|.
An initial disturbance $v^2 \bar{g}_2$ introduced into a homogeneously prestressed nonlinear elastic plate propagates along the quasi-transveral MONGE-cone, mainly |2,3|. Since there are no pure modes, the contour lines in figs. 1 show also the component $v^1 \bar{g}_1$ propagating along the quasi-longitudinal cone. It is one order of magnitude smaller. The calculated wave fronts have been drawn into figs. 1 for comparison.
Using a sufficiently refined mesh STONELY and RAYLEIGH waves with steep gradients and high energy concentration in a small surface layer can be calculated. The sequence of figs. 2 shows the contour lines of principle shear stresses for a forming RAYLEIGH wave excited by an explosion on the free surface of an isotropic linear elastic plate. Figs. 3 show the contour lines and a perspective view of shear stresses for a fully developed RAYLEIGH wave. The fully developed wave agrees very well with the typical experimental pattern |8|. The smoke from the explosion and its absorption do not allow a quantitative comparison of the forming of the experimental RAYLEIGH wave with computation, so far.

The numerical results confirm the suspicion of experimentalists that the details of RAYLEIGH waves may strongly depend on the not exactly known pressure pulse form of the explosion. Encouraged by the good agreement with experiments we hope to support experiments by numerical simulations and thus to colour some blanc areas in observations. As an example only little is known about the physical processes that cause the transition from quasi-static fracture to the formation of shear bands at a crack-tip |9|. From numerical calculation more details about stress wave interaction with crack-tips are expected.

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