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Direct static FEM approach to limit and shakedown analysis

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Abstract. *Safety and reliability of structures may be assessed indirectly by stress distributions. Limit and shakedown theorems are simplified but exact methods of plasticity that provide safety factors directly in the loading space. These theorems may be used for a direct definition of the limit state function for failure by plastic collapse or by inadaptation. In a FEM formulation the limit state function is obtained from a nonlinear optimization problem. This direct approach reduces considerably the necessary knowledge of uncertain technological input data, the computing time, and the numerical error. Moreover, the direct way leads to highly effective and precise reliability analyses. The theorems are implemented into a general purpose FEM program in a way capable of large-scale analysis.*

1 Introduction

The elastic strain range of pressure vessels and piping is rather small already in normal operation. Severe thermal loads or other residual stresses may push stress further into the plastic range locally. Therefore, all relevant design codes consider plastic deformation of ductile materials using the concepts of limit and shakedown analysis for design-by-formulae (DBF) or for stress assessment. This current design practice rests on simplifying assumptions for geometry, loading and constitutive equation.

A first improvement is achieved by inelastic finite element analyses. But stress assessment produces more questions than it can answer [6]. As a second more direct option the design-by-analysis (DBA) route of the new European unfired pressure vessels standard proposes the use of the direct methods of limit and shakedown analysis to compute the load carrying capacity [17]. Plastic design cannot be based on stress assessment, because there is no stress to bound the plastic range from failure domains. DBA considers the characteristic development of plastic strains towards structural failure:

- Instantaneous collapse by unrestricted plastic flow at limit load (gross plastic deformation).
- Incremental collapse by accumulation of plastic deformations over subsequent load cycles (ratchetting, progressive plastic deformation).
- Low Cycle Fatigue (LCF) by alternating plasticity.
- Plastic instability of slender compression members (buckling).

Limit and shakedown analyses deal directly with the first three of these failure evolutions, which are iconized in the Bree-Diagram (see Fig. 1 and [3]). Although being simplifying methods, they are exact theories of classic plasticity, which do not contain any restrictions or assumptions other than sufficient ductility of the material. The direct limit and shakedown analysis approach computes the load carrying capacity or the safety factor independently of the details of material behaviour and of the generally unknown load history [7], [13], [8].

Design and assessment of engineering structures imply decision making under uncertainty of the actual load carrying capacity of a structure. Uncertainty may originate from random fluctuations of significant physical properties, from limited information and from model idealizations of unknown credibility. Structural reliability analysis deals with all these uncertainties in a rational way. Direct DBA provides a well-defined limit state function for reliability analysis. In a finite element approximation this definition of a limit state may be combined with first order reliability analysis (FORM) for highly effective and robust computations of low failure probabilities. A first theoretical basis of the probabilistic approach is based on the static theorems of limit and shakedown analysis.

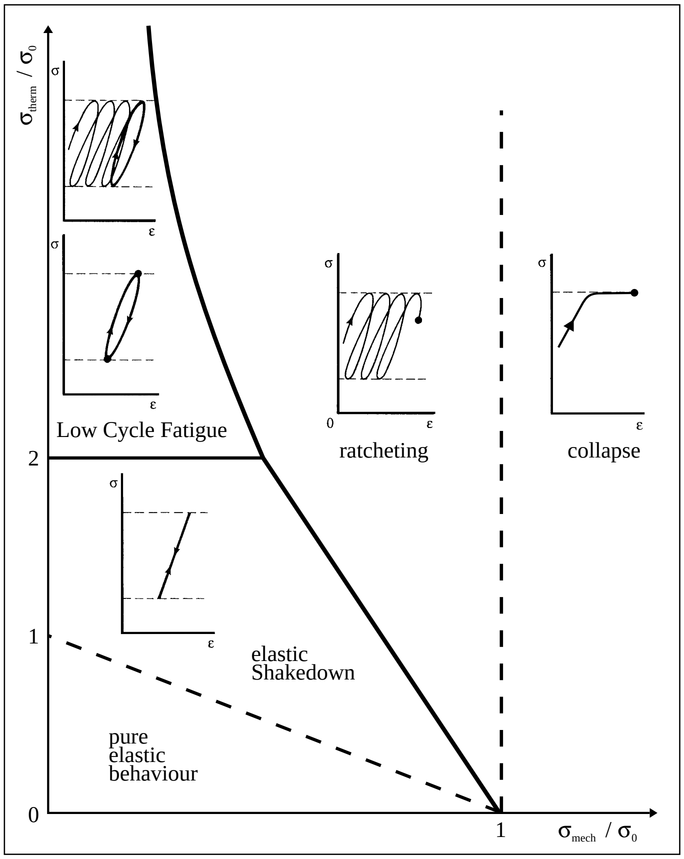


Figure 1: Bree-Diagram of thin wall tube

2 Reliability analysis

The behaviour of a structure is influenced by various typically uncertain parameters (loading type, loading magnitude, dimensions, material data,…). All parameters are described by random variables collected in the vector of basis-variables $\mathbf{X} = (X_1, X_2, ...)$. We will restrict ourselves to those basis–variables X_j which could be described by densities f_j, such that the joint density $f(x_1, \ldots, x_n)$ exists and the joint distribution function $F(\mathbf{x})$ is given by

$$F(\mathbf{x}) = P(X_1 < x_1, \ldots, X_n < x_n) \quad (1)$$

$$= \int_{-\infty}^{x_1} \cdots \int_{-\infty}^{x_n} f(t_1, \ldots, t_n) dt_1 \ldots dt_n. \quad (2)$$

The deterministic safety margin $R - S$ is based on the comparison of a structural resistance R and loading S (which is usually a local stress at a hot spot or in a representative cross-section). With R, S function of $\mathbf{X}$ the structure fails for any realization with non-positive limit state function

$$g(\mathbf{X}) = R - S. \tag{3}$$

The limit state $g(\mathbf{X}) = 0$ defines the limit state surface ∂V which separates the failure region V from the safe region. The failure probability P_f is the probability that $g(\mathbf{X})$ is non-positive, i.e.

$$P_f = P(g(\mathbf{X}) \leq 0) = \int\limits_V f_X(\mathbf{x}) d\mathbf{x}. \tag{4}$$

Usually, it is not possible to calculate P_f analytically. The computational effort of straight forward MCS increases quickly with reliability. Therefore any effective analysis is based on First and Second Order Reliability Methods [5] (FORM/SORM).

The basis-variables $\mathbf{X}$ are transformed into standard normal variables $\mathbf{U}$, such that g_u is the corresponding limit state function in $\mathbf{U}$–space. In FORM, a linear approximation V_F of the failure region V is generated, such that the limit state surface ∂V is approximated in a point $\mathbf{u}_0$ by

$$\partial V = \left\{ \mathbf{u} \;\middle|\; \boldsymbol{\alpha}^T \mathbf{u} + \beta = 0 \right\}, \tag{5}$$

with $|\boldsymbol{\alpha}| = 1$. If the limit state function is linear in the $\mathbf{U}$–space, then the failure probability P_f is exactly given by

$$\mathrm{P}_f = \mathrm{P}(\boldsymbol{\alpha}^T \mathbf{U} \leq -\beta) = \Phi(-\beta) \tag{6}$$

where Φ is the Gauss distribution function. The failure probability P_f depends only on the safety index β. In limit analysis the limit state function is linear in strength and loading. This favourable situation is preserved in $\mathbf{U}$–space, if both are normally distributed.

The design point $\mathbf{u}^*$ is the solution of the optimization problem

$$\beta = \min\{\mathbf{u}^T \mathbf{u} \;\mid\; g_u(\mathbf{u}) \leq 0\}. \tag{7}$$

Then P_f may be computed from the minimum distance of the limit state surface ∂V from origin in this $\mathbf{U}$–space. The design point is the point $\mathbf{u}^*$ on ∂V, which is the closest to the origin. Failure is most probable for data near the design point. The limit state function $g_u(\mathbf{U})$ is approximated by its linear Taylor series of a point $\mathbf{u}_0 \in \partial V$ to generate the iterative procedure

$$\mathbf{u}_{k+1} = \frac{\nabla_u g_u(\mathbf{u}_k)}{|\nabla_u g_u(\mathbf{u}_k)|^2} \left[\mathbf{u}_k^T \nabla_u g_u(\mathbf{u}_k) - g_u(\mathbf{u}_k) \right]. \tag{8}$$

as a search algorithm for $\mathbf{u}^*$. This poses a nonlinear optimization problem which needs the gradient of $g_u(\mathbf{u})$. The derivates are determined by

$$\nabla_u g_u(\mathbf{u}) = \nabla_u g(\mathbf{x}) = \nabla_x g(\mathbf{x}) \nabla_u \mathbf{x}. \tag{9}$$

If the deterministic structural problem is solved in a step-by-step iterative FEM analysis this gradient information is obtained from a sensitivity analysis, which consumes much computing time. Extension of this type of reliability analysis to plastic structural failure faces several already mentioned problems which are not present in linear elastic analysis: local stress has no direct relevance to plastic failure and structural behaviour becomes load-path dependent. No straight-forward $g(\mathbf{X})$ is obtained from standard incremental analysis if failure is assumed by plastic collapse, by ratcheting or by plastic shakedown (LCF). It is even more difficult to obtain the gradient of $g(\mathbf{X})$. Therefore, as an additional draw-back MCS (improved by importance sampling or by some other means of variance reduction) is used in connection with incremental nonlinear reliability analyses.

In FEM, limit and shakedown analysis are formulated as optimization problems. For these optimization problems the Lagrange multipliers of the solution are the strength and the load components of the gradient vector $\nabla_x g(\mathbf{x})$. Thus, no sensitivity analysis is necessary.

3 Concepts of limit and shakedown analysis

Static theorems are formulated in terms of stress and define safe structural states by giving an optimization problem for safe loads. The maximum safe load is the limit load avoiding collapse. Alternatively, kinematic theorems are formulated in terms of kinematic quantities and define unsafe structural states yielding a dual optimization problem for the minimum of limit loads. Any admissible solution to the static or kinematic theorem is a true lower or upper bound to the safe load, respectively. Both can be made as close as desired to the exact solution. If upper and lower bound coincide, it could be stated that the true solution has been found.

3.1 Static or lower bound limit load analysis

The limit load factor is defined in (10) by $\alpha = \mathbf{P}_l / \mathbf{P}_0$, where $\mathbf{P}_l = (\mathbf{b}_l, \mathbf{p}_l)$ and $\mathbf{P}_0 = (\mathbf{b}_0, \mathbf{p}_0)$ are the plastic limit load and the chosen reference load, respectively. Here we have supposed that all loads (**b** body forces and **p** surface loads) are applied in a monotone and proportional way. We look for the maximum load factor for which the structure is safe. The structure is safe against plastic collapse if there is a stress field $\boldsymbol{\sigma}$ such that the equilibrium equations are satisfied and the yield condition is nowhere violated. We obtain the following maximum problem:

$$
\begin{aligned}
\max \quad & \alpha \\
\text{s. t.} \quad & \Phi(\boldsymbol{\sigma}) \leq \ \sigma_y \quad \text{in } \Omega \\
& \mathrm{div}\boldsymbol{\sigma} = -\ \alpha \mathbf{b}_0 \quad \text{in } \Omega \\
& \boldsymbol{\sigma}\, \mathbf{n} = \ \alpha \mathbf{p}_0 \quad \text{on } \partial\Omega_\sigma
\end{aligned}
\tag{10}
$$

for the structure Ω, traction boundary $\partial\Omega_\sigma$ (with outer normal $\mathbf{n}$), yield function Φ, body forces $\alpha\mathbf{b}_0$ and surface loads $\alpha\mathbf{p}_0$.

3.2 Static or lower bound shakedown analysis

The shakedown analysis starts from Melan's lower bound theorem [10]. In the shakedown analysis the equilibrium conditions and the yield criterion for the actual stresses have to be fulfilled at every instant of the load history. We look for the maximum load factor for which the structure is safe. The structure is safe against LCF or ratcheting if there is a stress field $\boldsymbol{\sigma}(t)$ such that the equilibrium equations are satisfied and the yield condition is nowhere and at no instant t violated. We formulate the next maximum problem:

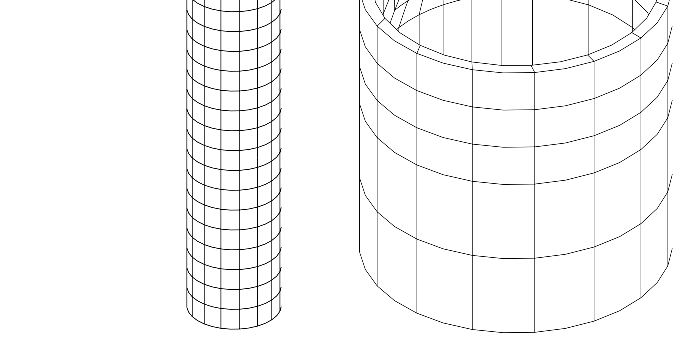
$$\begin{aligned} \max \quad & \alpha \\ \text{s. t.} \quad & \Phi(\boldsymbol{\sigma}(t)) \leq \ \sigma_y \quad \text{in } \Omega \\ & \text{div}\boldsymbol{\sigma}(t) = -\ \alpha \mathbf{b}_0(t) \quad \text{in } \Omega \\ & \boldsymbol{\sigma}(t)\,\mathbf{n} = \ \alpha \mathbf{p}_0(t) \quad \text{on } \partial\Omega_\sigma \end{aligned} \tag{11}$$

for body forces $\alpha\mathbf{b}_0(t)$ and surface loads $\alpha\mathbf{p}_0(t)$, for all $\mathbf{b}_0(t)$, $\mathbf{p}_0(t)$ in a given initial load domain $\mathcal{L}_0$.

In the maximum problems (10) and (11), the actual stresses $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}(t)$ are splitted into fictitious elastic and residual stresses. The deduced problem is solved by a basis reduction technique [7], [13], [15] in the residual stress space and by Sequential Quadratic Programming (SQP).

3.3 Pressurized pipe-junction subjected to additional bending load

The first example is a pipe-junction subjected to constant internal pressure P and an additional bending force F. The bending force is originated by an additional weight at the end of the nozzle operating in the shown direction.



(a) FEM-model (whole and part)

inner pipe radius	R	22 mm
inner nozzle radius	r	1.5 mm
pipe thickness	D	2 mm
nozzle thickness	d	3 mm
pipe length	L	250 mm
nozzle length	l	150 mm

(b) Dimensions of the pipe-junction

Figure 2: FEM-mesh and dimensions of the pipe-junction

Because of the symmetry condition only one half of the junction (and the weight load) is modeled. It is discretized with 924 solid 20-node hexahedron elements (PERMAS elements HEXEC20). The simple FE-mesh with only one element across the wall thickness, the dimensions and the material data of the pipe-junction are presented in Fig. 2(a) and 2(b). The material data are yield stress $\sigma_y = 250 N/mm^2$, Young's modulus $E = 2.1 \cdot 10^5 N/mm^2$ and Poisson ratio $\nu = 0.3$.

The pipe-junction subjected only to internal pressure starts to yield at a constant pressure of $P_y = 13.2$ MPa and a numerical lower limit of the collapse pressure is obtained as $P_{lim} = 23.5$ MPa. The corresponding undisturbed pipe has an analytical limit pressure of $P^r_{lim} = 25$ MPa (see [16]). This means that the junction has only a weakening effect of 6 %, such that the failure of the large pipe dominates the failure of the pipe-junction subjected to internal pressure.

If only the bending force F is acting, simple beam theory predicts first yield at $F_y = 117.8N$. The analytical limit load factor of a tube is

$$\alpha_{lim} = \frac{16}{3\pi} \cdot \frac{1 - (\frac{r_i}{r_o})^3}{1 - (\frac{r_i}{r_o})^4}. \tag{12}$$

With inner and outer tube radius $r_i = r$ and $r_o = r + d$ for the nozzle $\alpha_{lim} = 1.655$ and the limit load is $F_{lim} = 190.2N$. The numerical limit load is $F_{lim} = 175.2$. The difference could be addressed to the weakening effect of the geometrical discontinuity of the pipe-junction or to the coarse mesh.

If a constant internal pressure of 7 MPa is applied as a dead load and the bending force F is monotonically increased, the pipe-junction starts to yield at the nozzle corner at a bending force of $F_y = 49.2N$ or the additional weight of 4.9 kg. The limit load for the applied bending force calculated by PERMAS-LISA is $F_{lim} = 163.6N$ (16.7 kg), such that the limit load factor is $\alpha_{lim} = 3.34$.

In shakedown analysis the bending force varies between 0 and the maximum magnitude αF. The computed shakedown factor is $\alpha_{SD} = 2.52$, such that the maximal varying bending force is $F_{SD} = 123.4N$ (12.6 kg) (see Figure 3). The shakedown calculation becomes stationary after a few iteration steps. The shakedown pressure is twice the elastic pressure in good correspondence with an analytic solution see [7].

4 Implementation and test of reliability analysis

The present contribution uses lower bound theorems of limit and shakedown load to define a limit state function $g(\mathbf{X})$ for reliability analysis by FORM. R and S are respectively defined by the limit or shakedown load factor and the applied load factor. The commercial general purpose FEM code PERMAS [11] is used for discretization. The resulting large-scale optimization problem is transferred to a relatively small one by a basis reduction method [15], [13].

The solution of the limit load or of shakedown analysis (10) is a linear function of the failure stress σ_y or σ_u if a homogeneous material distribution is assumed. If the structure has a heterogeneous material distribution we obtain in different Gaussian points x_i eventually different

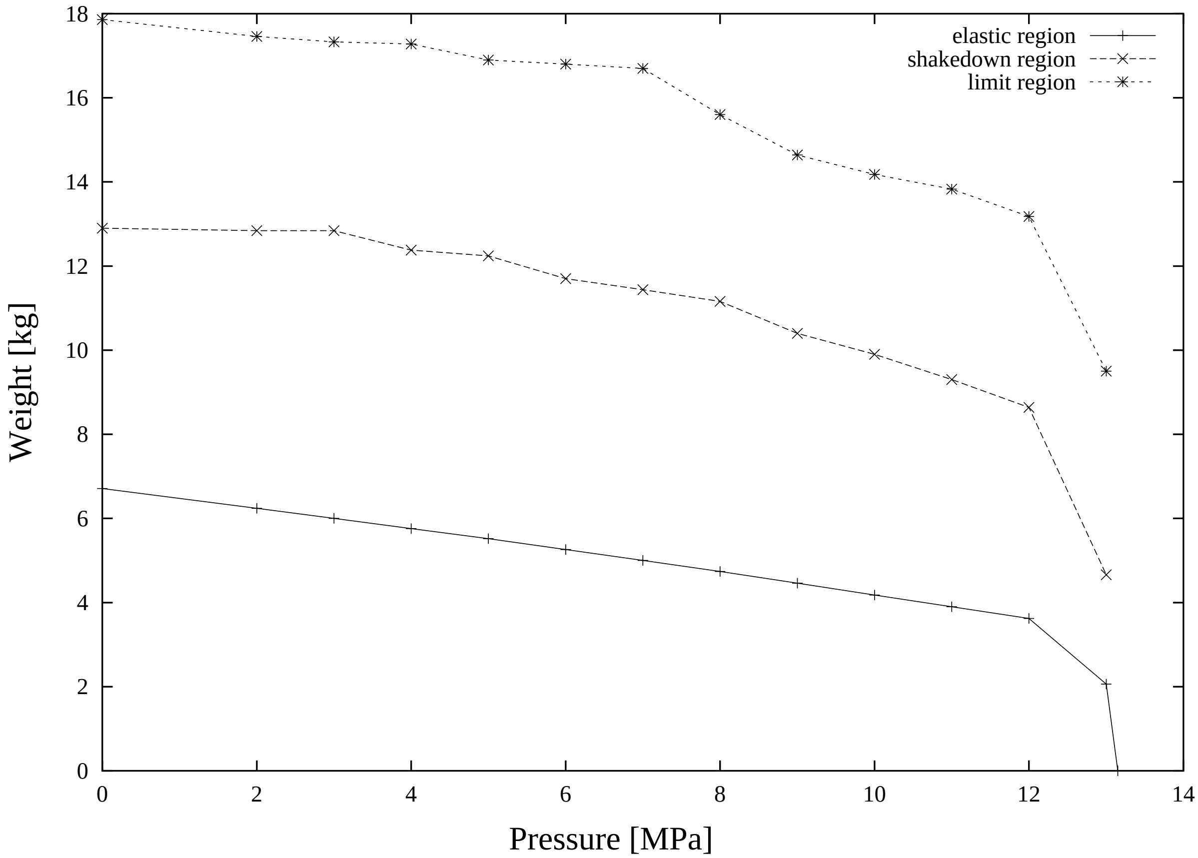


Figure 3: Interaction diagram of the pipe-junction under constant pressure and varying bending force

failure stresses $\sigma_y(x_i)$ or $\sigma_u(x_i)$. Then the limit load is no more a linear function of the failure stresses. In this case the derivates of the limit state function may not be computed directly from the linear function of the failure stresses. The Lagrange multipliers of the optimization problem (10) yield the gradient information of $g(\mathbf{X})$ without any extra computation. This is derived from a distribution theory of $\sigma_u(x_i)$ as the right hand side of (10) see [7].

It is most important for the analysis under uncertainty that limit and shakedown analyses are based on a minimum of information concerning the constitutive equations and the load history. This reduces the costs of the collection of statistical data and the need to introduce stochastic models to compensate the lack of data. Due to the so-called tail sensitivity problem there is generally insufficient data to analyze structures of high reliability which are e.g. employed in nuclear reactor technology. Probabilistic limit and shakedown analyses were pioneered in Italy [1]. Further work seemed to remain restricted to stochastic limit analysis of frames based on linear programming [2]. The present contribution extends plastic reliability analysis towards nonlinear programming, shakedown, and a general purpose large-scale FEM approach.

Typical structural components demonstrate that reliability analysis can be performed for realistic model sizes at very low computing times compared to incremental analyses. Note, that the latter cannot be used in a quantitative comparison because incremental nonlinear analysis fails to give a sharp evidence for plastic failure.

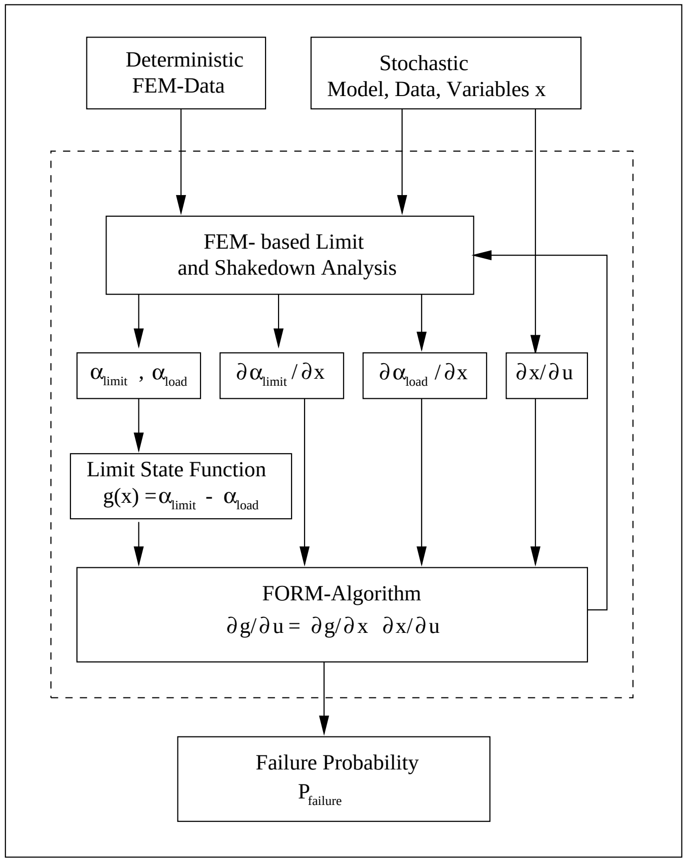


Figure 4: Flowchart of the probabilistic limit load analysis

4.1 Reliability calculations for limit analysis

In case of a square plate of length L with a hole of diameter D (see Figure 5) and $D/L = 0.2$ subjected to uniaxial tension the exact limit load is given by $R = (1 - D/L)\sigma_y$ with the yield stress σ_y (see [4], [14]).

Thus the limit load R depends linearly of the realization σ_y of the yield stress basic variable X. The load S is a homogeneous uniaxial tension on one side of the plate. The magnitude of the tension is the second basic variable Y. The limit load P_L of every realization y of Y is

$$P_L(y) = (1 - D/L)\ y. \tag{13}$$

The limit state function is defined by

$$g(x, y) = P_L - P = (1 - D/L)\, y - x. \tag{14}$$

As examples for non-linear distributions, log-normally distributed loads X and failure stresses Y are investigated. The density of log-normally distributed random variables with the parameters m, δ is given by

$$f(x) = \frac{1}{x\sqrt{2\pi\delta^2}}\, e^{-[\log(x/m)]^2/2\delta^2} \text{ , with } m > 0, x \geq 0. \tag{15}$$

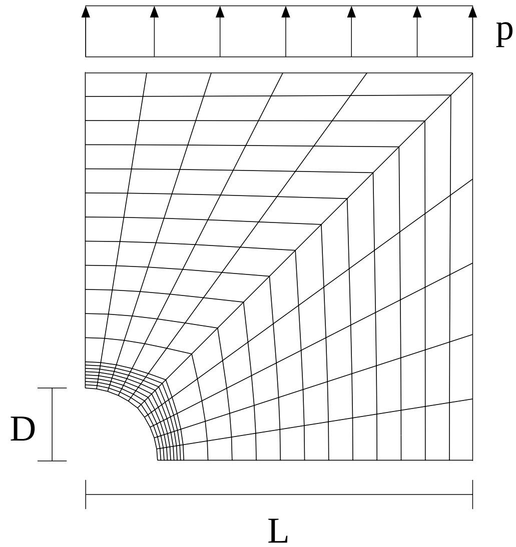


Figure 5: Finite element mesh of plate with a hole

The log-normal distribution has the expectation μ and the variance σ^2

$$\mu = E(\mathrm{X}) = m\, e^{\delta^2/2}, \qquad \sigma^2 = \mathrm{Var}(\mathrm{X}) = m^2\, e^{\delta^2}(e^{\delta^2} - 1) \tag{16}$$

For the comparison of the different random distributions the same expectation $\mu_{x,y}$ and variance $\sigma^2_{x,y}$ must be chosen, such that the values of $\mu_{x,y}$ and $\sigma_{x,y}$ have to be transformed to the parameters $m_{x,y}$ and $\delta_{x,y}$:

$$m_{x,y} = \mu_{x,y}\, e^{-\delta^2_{x,y}/2} \qquad \text{and} \qquad \delta_{x,y} = \sqrt{\log\left(\frac{\sigma^2_{x,y}}{\mu^2_{x,y}} + 1\right)}. \tag{17}$$

If X and Y are log-normally distributed the random variables $\bar{\mathrm{X}}$ = log (X) and $\bar{\mathrm{Y}}$ = log (Y) are normally distributed with means $\bar{\mu}_{x,y} = \log(m_{x,y})$ and deviations $\bar{\sigma}_{x,y} = \delta_{x,y}$, such that the following transformation holds

$$\log(x) = u_x\bar{\sigma}_x + \bar{\mu}_x = u_x\delta_x + \log(m_x) \tag{18}$$
$$\log(y) = u_y\bar{\sigma}_y + \bar{\mu}_y = u_y\delta_y + \log(m_y) \tag{19}$$

The transformation from X-space to U-space is nonlinear. The failure domain V is given by

$$V = \left\{\frac{(1 - D/L)\mathrm{Y}}{\mathrm{X}} \leq 1\right\} = \{\log(1 - D/L) + \log(\mathrm{Y}) - \log(\mathrm{X}) \leq 0\} \tag{20}$$

with the limit state function

$$g(\mathrm{X},\mathrm{Y}) = \log(1 - D/L) + \log(\mathrm{Y}) - \log(\mathrm{X}). \tag{21}$$

With the transformation we derive

$$g(\mathrm{X},\mathrm{Y}) = u_y\delta_y - u_x\delta_x + \log(1 - D/L) + \log(m_y) - \log(m_x), \tag{22}$$

such that β is given by

$$\beta = \frac{\log\left((1 - D/L)m_y\right) - \log(m_x)}{\sqrt{\delta_y^2 + \delta_x^2}} \tag{23}$$

In Table 1 it is shown that the results of the reliability analysis correspond very well with the analytical values.

Limit load analysis $\sigma_r = 0.1\mu_r$, $\sigma_s = 0.1\mu_s$			
μ_r/μ_s	P_f (num.)	P_f (anal.)	P_f (anal.-3%)
0.3	9.593E-12	1.790E-12	8.091E-12
0.4	1.409E-06	4.473E-07	1.316E-06
0.5	1.009E-03	4.315E-04	9.172E-04
0.6	3.485E-02	2.071E-02	3.412E-02
0.7	2.409E-01	1.719E-01	2.324E-01
0.8	5.936E-01	5.000E-01	5.854E-01
0.9	8.575E-01	7.981E-01	8.533E-01
1.0	9.648E-01	9.431E-01	9.638E-01
1.1	9.935E-01	9.880E-01	9.933E-01
1.2	9.990E-01	9.979E-01	9.989E-01
1.3	9.998E-01	9.997E-01	9.998E-01
1.4	9.999E-01	9.999E-01	9.999E-01
1.5	9.999E-01	9.999E-01	9.999E-01

Table 1: Numerical and analytical results for $\sigma_{r,s} = 0.1\mu_{r,s}$ (Log-normal distributions)

4.2 Reliability calculations for shakedown analysis

In the shakedown analysis a convex load domain $\mathcal{L}$ is analyzed [9]. The tension p cycles between zero and a maximal magnitude of p_0. Only the amplitudes but not the uncertain full load history enters the solution.

$$0 \leq \quad p \leq \quad \alpha\lambda p_0, \qquad 0 \leq \lambda \leq 1. \tag{24}$$

In the first simple reliability analysis the maximal magnitude p_0 is a random variable, but the minimum magnitude zero is held constant. the random variables are both normally distributed. The results of the FORM calculation are compared with an analytical approximation of the shakedown load in Table 2. For comparison the reliability analysis of limit analysis for normally distributed variable is also shown in Table 2.

Because of the local failure of the plate in the ligament points of the hole, the shakedown factor α_{SD} corresponding to the initial yield load p_i is equal to 2 (see [7], [18]). Therefore, from the

Limit load analysis				Shakedown analysis		
μ_r/μ_s	P_f (num.)	P_f (anal.)	P_f (anal.-2%)	μ_r/μ_s	P_f (num.)	P_f (anal.)
0.2	2.643E-13	1.718E-13	2.640E-13	0.2	1.943E-10	1.943E-10
0.3	3.843E-09	2.426E-09	4.063E-09	0.3	5.964E-06	5.963E-06
0.4	6.112E-06	3.872E-06	6.416E-06	0.4	3.877E-03	3.877E-03
0.5	1.093E-03	7.364E-04	1.128E-03	0.5	1.227E-01	1.229E-01
0.6	3.049E-02	2.275E-02	3.118E-02	0.55	3.108E-01	3.111E-01
0.7	2.067E-01	1.734E-01	2.112E-01	0.59	5.000E-01	5.000E-01
0.8	5.550E-01	5.000E-01	5.567E-01	0.6	5.485E-01	5.485E-01
0.9	8.305E-01	7.969E-01	8.344E-01	0.65	7.538E-01	7.538E-01
1.0	9.544E-01	9.408E-01	9.554E-01	0.7	8.858E-01	8.858E-01
1.1	9.900E-01	9.863E-01	9.903E-01	0.8	9.828E-01	9.828E-01
1.2	9.981E-01	9.972E-01	9.981E-01	0.9	9.980E-01	9.980E-01
1.3	9.996E-01	9.995E-01	9.996E-01	1.0	9.997E-01	9.997E-01
1.4	9.999E-01	9.999E-01	9.999E-01	1.1	9.999E-01	9.999E-01

Table 2: Comparison of numerical and analytical results for $\sigma_{r,s} = 0.1\mu_{r,s}$ (Normal distributions)

yield load $p_i = 0.2949\sigma_y$ resulting from the deterministic FEM-computation follows that the FEM–approximation of the shakedown load is $0.5897\sigma_y$. The implemented shakedown analysis with the basis reduction technique gives very good results for the reliability analysis of the plate (listed in Table 2), because the deterministic shakedown factor 2 is reached in 3 to 5 steps nearly identically.

Additionally, the shakedown reliability analysis needs less computing time than the limit load reliability analysis. The results of the shakedown reliability analysis show a decrease in reliability in comparison with the limit load reliability results. For a load level of $0.4\mu_r$ the reliability decrease by 3 orders of magnitude. This means that the reliability of the structure depends very strongly on the loading conditions, such that the assessment of the load carrying capacity has to be done very carefully.

5 Conclusions

As limit and shakedown analysis deals directly with the failure modes, the results give better insight for the designer into the structural behaviour under all possible mechanical or thermal actions. These direct methods of plastic structural failure analysis provide direct definitions of limit state functions. In combination with FEM and with FORM, low failure probabilities of passive components are obtained with sufficient precision at low computational efforts. Sensitivities need no extra FEM analysis, because the gradient of the limit state function is also obtained as a by-product of FEM limit and shakedown analysis. The remaining numerical error may be estimated or reduced by the additional use of upper bound theorems. Further research is also addressed to more realistic material modeling including non-linear hardening and continuum damage.

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