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DESIGN BY ANALYSIS OF PRESSURE COMPONENTS BY NON–LINEAR OPTIMIZATION

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ABSTRACT
This paper presents the direct route to Design by Analysis (DBA) of the new European pressure vessel standard in the language of limit and shakedown analysis (LISA). This approach leads to an optimization problem. Its solution with Finite Element Analysis is demonstrated for some examples from the DBA–Manual. One observation from the examples is, that the optimisation approach gives reliable and close lower bound solutions leading to simple and optimised design decision.

INTRODUCTION
In the new European standard for unfired pressure vessels, EN 13445-3, [4], there are two approaches for a Design-by-Analysis (DBA) for sufficiently ductile steels and steel castings below the creep range. They cover both the stress categorization method (Annex C) and the direct route method (Annex B). The stress categorization method is an elastic route to the assessment of inelastic structural failure. Conceptually this stress categorization originates from limit and shakedown analysis (LISA) for simple beam and thin shell structures. It can hardly be recommended for more complex geometries [3], [6].

The new direct route by elasto-plastic calculation in EN 13445-3 Annex B [4] seems to be the more promising alternative. The paper relates the direct route in [3] with advanced limit and shakedown analysis (LISA). The work has been performed in the European research project LISA between January 1998 and May 2002.

DESIGN CHECKS
The direct route calculates the design resistance (limit action) with respect to ultimate limit states of the structure. Design checks are designated by failure modes. The following ones are included in the first issue of EN 13445-3 Annex B [4], [3] :

- Gross Plastic Deformation (GPD), with excessive local strains and ductile rupture (collapse).
- Progressive plastic Deformation (PD), with incremental collapse (incremental collapse, ratchetting).
- Instability (I), with large displacements to a new stable geometry of the structure under compressive actions (buckling).
- Fatigue (F), with alternating plasticity (AP) or with high cycle fatigue.
- Static Equilibrium (SE), with possible overturning and rigid body movement.

Actions denote in [4] all thermo-mechanical quantities imposed on the structure causing stress or strain. Actions are classified by their variation in time: permanent (G), variable (Q), exceptional (E), and operating pressures and temperatures (**p**, T).

LOWER BOUND ANALYSIS
Traditionally the GDP check has to be performed with the more conservative Tresca yield surface $\Phi_T(\sigma)=0$, whereas the PD check uses the more realistic von Mises yield surface $\Phi_M(\sigma)=0$. The numerical difficulties in FEA with the non-smooth Tresca yield surface are avoided in [3] by reducing the von Mises limit by $\sqrt{3}/2$. This strict use of the Tresca rule is very conservative for spherical shells. Therefore [3] does not strictly follow the code principle in EN 13445-3 B.8.2.1 [4] for the GDP check for such shells.

Check against failure modes GPD and PD may be directly performed by LISA based on Melan's static or lower bound theorem. Conservatively the code [4] requires the use of a perfectly plastic material model and the side-condition that the maximum absolute value of the principle strains does not exceed 5%. It could be asked if the simple perfectly plastic model allows a conservative estimation of the plastic strain accumulation during ratchetting, because all known cyclic plasticity models fail on one or more material ratchetting experiments [2]. However, it is easily demonstrated that the difference in shakedown analyses for some linear and nonlinear kinematic hardening models is smaller than could be expected from cyclic analyses with such models [19].

The shakedown check includes a check against AP, which is not required in a PD check [4]. This is accepted in the DBA manual ([3], p. 2.40), because it is conservative and because the proof of shakedown is easier to perform than cyclic plastic analyses. Shakedown analysis can distinct between AP and PD [21].

CHECK AGAINST GLOBAL PLASTIC DEFORMATION (GPD)

Static theorems are formulated in terms of stress. They define safe structural states leading to an optimization problem for safe loads. The maximum safe action is respectively the limit load (avoiding GPD) and the elastic shakedown load (avoiding PD and AP). Let us assume that the most unfavourable actions have been combined to a single design action $E_d = (\mathbf{q}_d , \mathbf{p}_d)$ with forces $\mathbf{q}_d$ in volume V, and vector surface traction $\mathbf{p}_d$ on the traction boundary ∂V_σ with unit normal vector **n**. Then - in the sense of the code - the action is plastically admissible if the yield condition

$$\Phi_T(\sigma) \le R_d \text{ in } V$$

is satisfied with the design resistance R_d (allowable stress). Trivially the structure must be in equilibrium, i.e.

$$-\text{div}\, \sigma = \alpha_S\, \mathbf{q}_d \quad \text{in } V$$
$$\mathbf{n}^T \sigma = \alpha_S\, \mathbf{p}_d \quad \text{on } \partial V$$

These conditions can be stated in words as the

Static limit load theorem:

An elastic-plastic structure will not collapse (GPD check) under a monotone effect $\sigma = \alpha_S\, \mathbf{p}_d\, E_d$, *if it is in static equilibrium and if the yield condition is nowhere violated.*

For each stress field σ, which fulfils the conditions of the static theorem, α_S is a safety factor, so that the design check is the requirement $\alpha_S \ge 1$.

One is interested in the largest factor, for which the structure does not collapse. The stress σ can be decomposed in the elastically calculated stress σ^E and some time independent stress ρ i.e. $\sigma(\mathbf{x}, t) = \sigma^E(\mathbf{x}, t) + \rho(\mathbf{x})$. Clearly, the stress field $\rho(\mathbf{x})$ is self-equilibrated, because σ and σ^E are in equilibrium with the same loading. In this notation the theorem leads to the infinite optimization problem

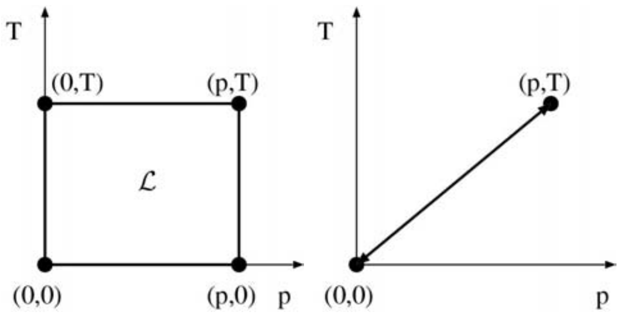
$$\max \alpha$$
$$\text{such that} \quad \Phi_T(\sigma^E(\mathbf{x}, t) + \rho(\mathbf{x})) \le R_d(\mathbf{x}) \quad \text{in } V$$
$$-\text{div}\, \rho = \mathbf{0} \quad \text{in } V$$
$$\mathbf{n}^T \rho = 0 \quad \text{on } \partial V$$

With the Tresca criterion $\Phi_T(\sigma)$ this is a linear optimisation problem. In the LISA project only nonlinear optimisation problems are considered with the von Mises criterion $\Phi_M(\sigma)$. The GPD check is performed by limit analysis with monotonously increasing action. This maximization problem is discretized by FEM and solved effectively with optimization algorithms [15], [18]. A large α_S can be used to improve the design. Additionally, the structural strains shall be limited to 5%. GPD is connected with loss of structural stiffness. Therefore the effect of strain limits on the limit action is often smaller than the numerical errors in practice.

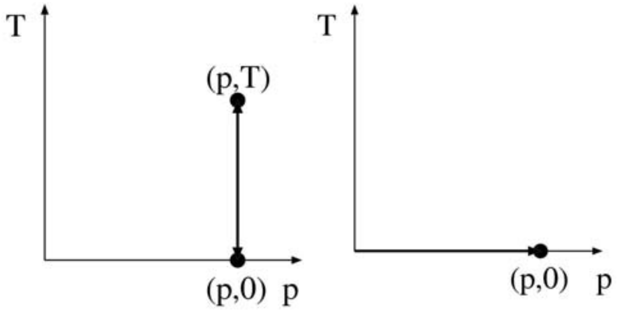
It should be clearly stated that, without restrictions, the limit load is truly independent of load history, elastic data and self-equilibrated stress (secondary stress). This statement is slightly modified if the side-condition on the maximum absolute value (5%) of the principle strains is considered.

CHECK AGAINST PROGRESSIVE PLASTIC DEFORMATION (PD)

Strictly, only one load case can be checked by limit analysis. Design check should be performed by shakedown analysis, if actions are not monotone. The time history of an action $\mathbf{A}_j(t) = (\mathbf{q}_j(t), \mathbf{p}_j(t), T_j(t))$ is often not well-known. It can however usually be stated that the actions vary within given amplitudes or admissible bounds. They define a convex load range $\mathcal{L}$.



Two-parameter action: NV=4	One-parameter action with proportional loading: NV=2



One-parameter action with dead load: NV=2	Limit analysis, proportional loading: NV=1

Tab. 1: Different loading cases.

If NV is the number of independent actions $\mathbf{A}_1,\ldots,\mathbf{A}_j,\ldots,\mathbf{A}_{NV}$ then all actions $\mathbf{A}(t)$ in $\mathcal{L}$ can be represented by a convex combination of NV vertices $\mathbf{A}_j$ of $\mathcal{L}$ with $\lambda_j(t) \ge 0$, and

$$\mathbf{A}(t) = \lambda_1(t)\, \mathbf{A}_1 + \ldots + \lambda_j(t)\, \mathbf{A}_j + \ldots + \lambda_{NV}(t)\, \mathbf{A}_{NV}, \quad \sum_{j=1}^{NV} \lambda_j = 1$$

The load-carrying capacity is exhausted by enlargement of $\mathcal{L}$ with the factor $\alpha_S > 1$ causing PD, AP or GPD. The shakedown theory analyses only the shakedown state. The shakedown theorems answer the question, whether a structure from ductile material is plastically safe or not. Generally, a structure under a load range $\mathcal{L}$ shakes down, if for each load in $\mathcal{L}$ an admissible stress field can be found, which is in equilibrium with this load; in other words

Static shakedown theorem:
An elastic-plastic structure will not fail with macroscopic plasticity (PD, AP, and GPD as special case) under time variant actions in $\alpha_S \mathcal{L}$ if it is in static equilibrium, if the yield function is nowhere and at no instance violated. Then the plastic deformation rates tend to zero.

Strictly, an independent fatigue analysis must show that the plastic dissipation is bounded. Again one is interested in the largest factor α_S, for which the structure shakes down to asymptotically elastic behaviour.

The same conditions as in the limit load theorems must be satisfied simultaneously at all times. Their examination in infinitely many instants is impossible and in addition, unnecessary. One can show that it is sufficient to satisfy the shakedown conditions only in the NV basis actions $\mathbf{A}_1,\ldots,\mathbf{A}_j,\ldots,\mathbf{A}_{NV}$ of $\mathcal{L}$ since the shakedown theorems lead to convex optimization problems. With the decomposition $\sigma_j(\mathbf{x}) = \sigma^E{}_j(\mathbf{x}) + \rho(\mathbf{x})$ we have:

$$
\begin{aligned}
&\max \alpha \\
&\text{such that} \quad \Phi_M(\alpha_S \sigma^E{}_j(\mathbf{x}) + \rho(\mathbf{x})) \le R_d(\mathbf{x}) \quad \text{in V, } j=1,\ldots,NV \\
&\qquad\qquad -\text{div}\, \rho(\mathbf{x}) = \mathbf{0} \quad \text{in V} \\
&\qquad\qquad \mathbf{n}^T \rho(\mathbf{x}) = 0 \quad \text{on } \partial V
\end{aligned}
$$

The optimisation problem is obtained with the only change that now the constraints have to be satisfied for all j=1,…,NV simultaneously. It is not sufficient to examine the critical load cases independently, because the shakedown analysis of $\mathcal{L}$ and the limit analysis of the critical load cases (vertices of $\mathcal{L}$) may give different results. The GPD check by limit analysis is included as the special load domain $\mathcal{L}$ with only one vertex, i.e. NV =1.

It should be pointed out that different to limit analysis, shakedown analysis does not apply if the elastically calculated stress field contains singularities. Therefore the standard [4] (Annex B.3.9.3.2) states that the check against PD can be performed for a stress-concentration-free structure. Sharp corners have to be removed from the FEM models, if they lead to unphysical peak stress.

DIRECT DESIGN CHECKS BY LISA

The shakedown optimization problem has been discretized with the Finite Element Method (FEM) using the general purpose FE code PERMAS [10] (Intes GmbH, Germany). The yield condition is checked in all Gaussian points. The main technical problem is the number of unknowns and restrictions in the discretized optimization problem. This has been solved by a basis reduction method in the present PERMAS implementation [8], [18]. Some important extensions of the shakedown theory have been achieved in the LISA project [15], [22]:

- LISA for bounded kinematic hardening, [7], [19], [15],
- LISA including continuum damage, [5],
- Limit analysis (GPD check) for crack containing structures, [20],
- Koiter's kinematic theorem for upper bound analysis, [21],
- Plastic reliability analysis for uncertain data of materials and actions, [9], [22],
- Distinction between AP check and PD check, [21], [22].

The last achievement is useful for DBA, because the AP check is not mandatory for the PD check in EN 13445-3, [4].

Different methods for large-scale optimization have been developed by the research groups contributing to the LISA project [15], [22]:

- Basis reduction by plastic analyses and search for the maximum in a sequence of low dimensional subspace by Sequential Quadratic Programming (SQP).
- Dual upper and lower bound analysis of the full size problem by large-scale nonlinear programming methods based on a sequence of linear elastic analyses.
- Reformulation of the problem in the form of a Second Order Cone Programming problem (SOCP).
- Zarka's method has been contributed as an additional direct plasticity method, which estimates the plastic deformation prior to failure.

The methods avoid any cyclic analysis and are therefore extremely effective for PD checks. A typical shakedown analysis needs only 2-3 times the computing time for one elastic static analysis [18]. With a computing time of about 10-20 elastic analyses the limit analysis for the GPD check seems to be somewhat less effective. One of the main advantages of the LISA method is seen in the possibility to indicate convergence to the true limit and shakedown loads by comparing lower and upper bounds.

All implementations of shakedown analysis in FEM codes have been checked against analyses of the LISA project partners, analytical solutions, and published results. Today only the basis reduction method has been implemented in the commercial FEM code PERMAS [10]. The implementation of some of the other methods in general purpose FEM codes is in preparation.

This paper discusses some of the results obtained by Aachen University of Applied Sciences, Germany (AcUAS) for examples taken from the DBA Manual [3]. In particular the lower bound LISA which solves the optimisation problem with the basis reduction method and analytical cylinder solutions are compared to the following contributions to the DBA–Manual [3]:

- Design by Formula (DBF) by Sant' Ambrogio Servizi, Italy (St. A.),
- DBA with the deviatoric map (DRS) by the Vienna Technical University, Austria (A&AB), [14],
- Elastic compensation method (DRC) by the University of Strathclyde, UK, [11],
- Nonlinear step-wise finite element analysis (NL) by the University of Strathclyde, UK,

- Stress classification, stress categorization (SC) by the University of Strathclyde or by TKS, Sweden.

Only two examples are chosen for comparison due to space limitations. The elastic compensation method and the basis reduction method have been used to compute load interaction diagrams. All the analyses have been performed with von Mises material. The design code EN 13445-3, [4] requires Tresca material for the GPD check. Therefore the DBA–Manual [3] suggests the correction by the factor $2/\sqrt{3}$. The examples demonstrate that such a correction could be questioned.

The DBA–Manual [3] is mainly restricted to mechanical loading, because the elastic compensation method could not deal with thermal loading. This restriction has been removed meanwhile [13]. The methods involving non-linear static analysis and superposition for non-proportional loading in [12] have been developed also with the objective to overcome some restrictions of the elastic compensation method. All the optimization methods in the LISA project have been developed also for thermal loading [22]. The application to such problems has been demonstrated elsewhere [7], [18], [21], [22].

GPD DESIGN CHECK OF PIPE JUNCTION

A thick–walled cylinder–cylinder intersection under internal pressure and a constant moment M=711.1Nm action (dead load) on the thin nozzle is considered (Fig. 1). There is a strength mismatch between cylinder material P265GH according to EN 10028-2 with $RM=R_{p0.2}$ =234 MPa and nozzle material 11CrMo9-10 according to prEN 10216-2 with $RM=R_{p0.2}$ =343 MPa. The FEM discretization of the structure consists of 4312 eight nodes volume elements (PERMAS element HEXE8).

The limit load is independent from the load history. Therefore the same limit load interaction diagram (Fig. 2) is obtained for a constant moment or for any other loading. The diagram has been computed for von Mises material. It shows that the elastic compensation method did not converge to a good lower bound. This behaviour was generally observed with other examples in the DBA–Manual. The interaction diagram is used to discuss different ways to read a design pressure.

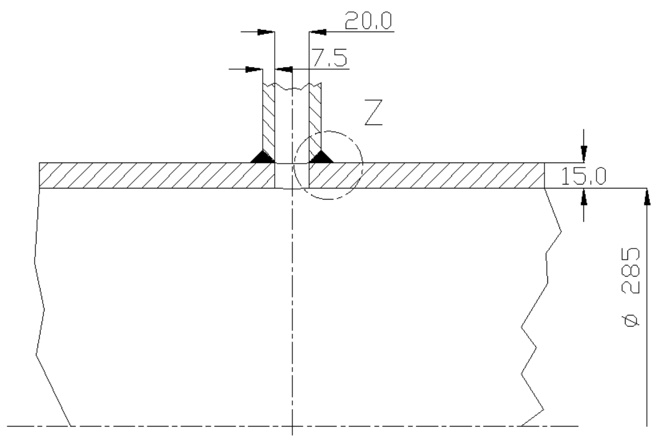


Fig. 1: Thick-walled cylinder –cylinder intersection.

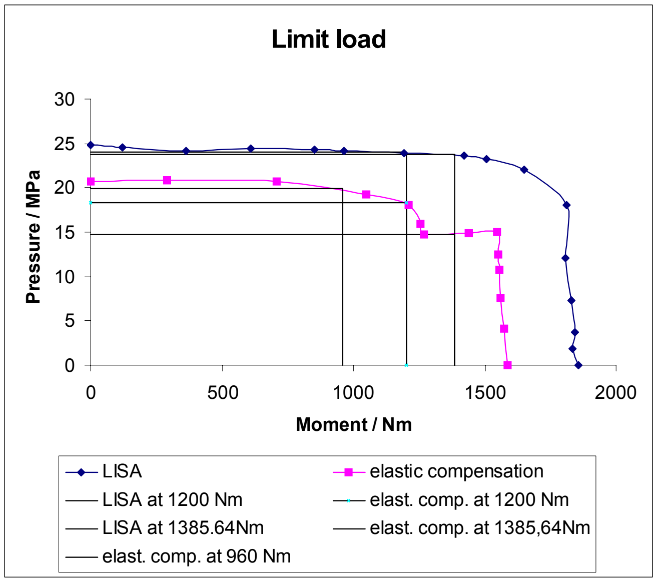


Fig.2: Limit load interaction diagram for thick-walled cylinder –cylinder intersection.

In the DBA Manual the limit pressure for moment dead load 711.1Nm has been determined by A&AB and by Strathclyde with only the partial safety factor γ_G=1.35 for the permanent moment action leading to the design moment 1.35·711.1Nm =960Nm. However, the partial safety factor γ_R=1.25 for the resistance should also be used such that the design moment is 1.35·1.25·711.1Nm=1200Nm. The tabulated design pressure is obtained by reduction with the partial safety factor γ_R=1.25, the partial factor γ_p=1.2 for pressure loading, and with the factor $\sqrt{3}/2$. If possible, the results from the DBA–Manual have been corrected accordingly for the Tresca material. They have been put in brackets in Tab. 2 to show that the tabulated values differ from the DBA–Manual.

Check		Project Member and Analysis Type						
		St. A.	A&AB	Strathclyde			AcUAS	
		DBF	DRS	DRC	NL	SC	LISA	analytic
Tresca limit load with safety factors		14.09	(14.79)	(11.5) 10.57	(15.1)	14.25	14.34	15.62
von Mises limit load without safety factors			(25.62)	(19.9) 18.3	(26.2)		24.84	27.04
Shakedown load	A	14.09	17.65	12.7	-	14.25	17.15	
	C	-	-	-	-	-	15.20	

Tab. 2: Comparison of the different approaches for the cylinder-cylinder intersection.

Strictly the design moment has to be converted to Tresca material to satisfy the EN 13445-3 requirements, [4]. The DAB–Manual suggests $2/\sqrt{3}$·1200Nm=1385.6Nm. This would reduce the elastic compensation (DRC) design pressure further to 8.5 MPa. The general use of a factor $2/\sqrt{3}$ may be not recommended here, because the bending stress in the nozzle is uniaxial. Then both yield functions give the same effective stress.

The exact limit pressure of the large cylinder is used for the analytic solution although the collapse occurs in the nozzle. The elastic compensation gives very conservative results below the DBF values.

A&AB has performed the limit analysis with a reduced design strength $\sqrt{3}/2 \cdot RM/\gamma_R = \sqrt{3}/2 \cdot RM/1.25$ to obtain the limit pressure 17.91MPa. This was connected with a calculated maximum principal strain of 14%. The limitation to 5% is required in EN 13445-3, [4]. This requirement is satisfied already for a limit pressure 17.75MPa. This small difference is below the observed numerical errors of the FEM analyses. The tabulated value is $17.75\text{MPa}/\gamma_p = 14.79\text{MPa}$.

Without pressure the limit moment is 1993.6Nm which gives the design value 1993.6Nm/(1.35·1.25)=1181.4Nm with the partial safety factors. Conservatively converting to Tresca material the design moment would be 1023.1Nm.

PD DESIGN CHECK OF PIPE JUNCTION

The same thick–walled cylinder–cylinder intersection is considered in a shakedown analysis. No safety factors shall be used in a PD check. The von Mises yield function applies. The interaction diagram (Fig. 3) is calculated for a dead load by a moment on the nozzle. It is denoted case A in Tab. 2 and it is obtained for a load domain L with NV=2. It is compared to the elastic compensation method. The latter gives again generally very conservative lower bounds. Additionally independent pressure and moment cycles are considered for a load domain L with NV=4 (two parameter loading, case C in Tab. 2).

As no safety factor is involved the design pressure is found for a moment of 711.1 Nm. The elastic compensation (DRC) value is close to the operating pressure cycles $P_{op,sup}$=12.68MPa so that the structure would be at the design limit. It is safe however with the other methods (A&AB and LISA).

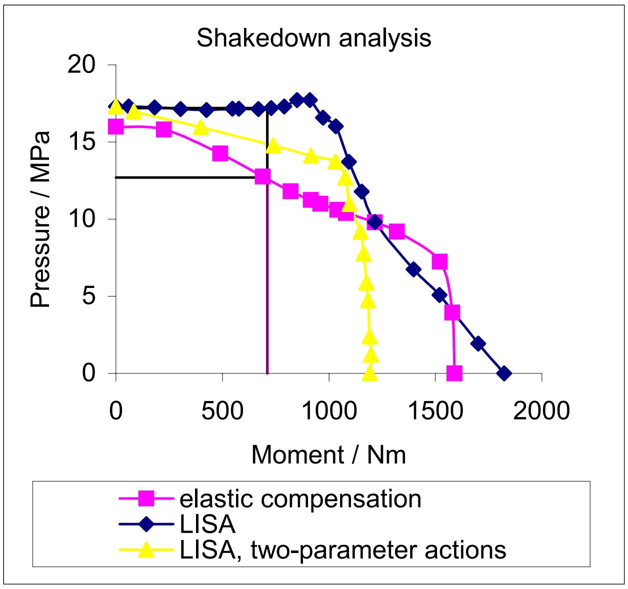


Fig.3: Limit load interaction diagram for thick-walled cylinder –cylinder intersection.

GPD DESIGN CHECK OF NOZZLE IN SPHERICAL END

A nozzle in a spherical shell with strength mismatch is considered under thermal and mechanical actions. The shell is made of 11CrMo9-10 according to EN 10028-2 with $RM=R_{p0.2}$=230MPa at 325°C. The nozzle reinforcement is made of 11CrMo9-10+Q according to prEN 10216-2 with, $RM=R_{p0.2}$=284MPa and the nozzle is made of P265 according to prEN 10216-2 with $RM=R_{p0.2}$=147.5MPa only.

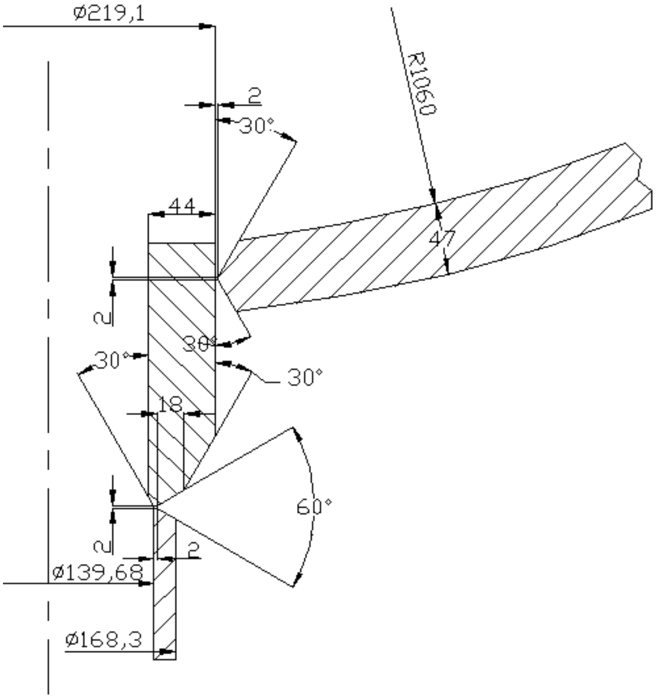


Fig.4: Nozzle in spherical end.

The limit analyses are collected in Tab.3. The values in brackets have been obtained indirectly by conversion between von Mises and Tresca yield function. The exact limit pressure of a spherical shell is given as the analytical solution. The exact solution holds for both, Tresca and von Mises Material. The FEM analysis shows that the spherical shell dominates the stress also in the nozzle region. Therefore A&AB has not used the conservative conversion from von Mises to Tresca yield condition in the DBA–Manual. This is not permitted formally by the code principle in EN 13445-3 B.8.2.1 [4]. However, one can bring up hardly technical reason for the formal penalization of the von Mises yield function in the GPD check for this analysis example.

Again, the elastic compensation is too conservative for most constant moment actions. For a smaller loading range the LISA optimisation approach needs also some improvement. All stepwise FEM analyses of perfectly plastic material suffer from some numerically induced kinematic hardening. This contributes only 2–3% excess strength This spurious kinematic hardening effect is absent in the LISA optimisation approach.

Criterion	St. A.	A&AB	Strathclyde		TKS		AcUAS	
	DBF	DRS	DRC	NL	SC ANSYS	SC BOSOR	LISA PERMAS	anal.
Tresca limit load with safety factors	13.01	(11.30)	11.1	(11.43)	12.8	11.7	(11.50)	13.31
von Mises limit load with safety factors		13.04	10.14	13.2			13.27	13.31
von Mises limit load without safety factors		19.56	17.57	19.8			19.91	19.96
Numerical hardening correction		1.027	1.021				1.00	-

Tab. 3: Comparison of the different approaches for the nozzle in spherical end.

CHOICE OF THE YIELD FUNCTION

Design codes for pressure vessels use different yield functions depending on the considered design check. The material however, is considered to follow only one yield function independent of the application. Generally the von Mises rule is considered to be the more realistic choice. It is also used for the PD check. The GPD check with Tresca yield function leads to a non-homogenous increase of the safety factor by 0–15.47% depending on the dominating stress state in the relevant failure region. There is no reason why some structure should need an additional safety measure (cylindrical shell) and other do not (spherical shell).

The recommendation of the DBA–Manual [3] to calculate with von Mises yield function and to convert always to Tresca yield condition afterwards, leads to a homogenous second hidden safety factor of $2/\sqrt{3}=1.155$ for all GPD checks. If this should really be considered to be necessary, it would simplify the code application if the von Mises condition would be used for all design checks and the partial safety factor γ_R would be modified to $2\gamma_R/\sqrt{3}$ to include the hidden safety factor for all structures in a GPD check. This very conservative approach has been applied only partly in the DBA–Manual.

SUMMARY AND CONCLUSIONS

This paper presents the direct route to Design by Analysis (DBA) of the new European pressure vessel standard in the language of limit and shakedown analysis (LISA). This approach leads to an optimization problem. Its solution with Finite Element Analysis is demonstrated for some examples from the DBA–Manual. This approach is particularly effective compared to cyclic non-linear analysis, because shakedown analysis needs typically the computing time of 2-3 elastic analyses. One observation from the examples is, that the optimisation approach gives reliable and close lower bound solutions leading a simple and optimised design decision. In combination with an upper bound approach the convergence to the true limits can be indicated. Some application rules have been critically discussed.

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