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OPTIMIZATION OF THE FLIGHT STYLE IN SKI JUMPING

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Abstract. During the flight phase the athlete has to optimize the aerodynamic forces in order to maximize the jump length while keeping the flight stable, both with respect to his features and abilities. A system of first order nonlinear differential equations describes the motion of a ski jumper and provides the basis for solving this constrained optimization problem with an optimization algorithm and comprehensive wind tunnel measurements. An optimization algorithm was developed on the basis of Pontryagin´s minimum principle combined with a penalty function derived from flight position constraints. By varying the constraints, it has was shown that there are various possibilities to reach comparable jump lengths and individual athletes can develop their individual optimum which is to be tuned with their personal features and abilities. In this study, the effect of the take-off velocity perpendicular to the ramp (v_{p0}) on the optimal flight style is examined. It is shown that v_{p0} has only minor effect on flight style optimization in elite ski jumping and a reference value of $v_{p0} = 2.5$ ms^{-1} can be used for further optimization studies. Optimization studies can be used advantageously for guiding the individual training. However, new comprehensive wind tunnel measurements with athletes using the latest equipment and field studies are necessary for more detailed optimization studies. The presented optimization approach can be applied to any sports which can be described by ordinary differential equations. This provides a useful basis for improving sports performance and equipment.

1 INTRODUCTION

Ski jumping is an Olympic discipline since the first Winter Games in 1924. International competitions are held on three types of hills differing in their hill size (HS), defined as the length between the edge of the take-off ramp and the end of the landing area: normal hill (85-109 m), large hill (109-184 m) and ski flying hill (>185 m)[1]. Including the four interrelated phases inrun, take-off, flight, and landing, ski jumping is technically very demanding since the athlete has to solve difficult optimization tasks within fractions of a second to reach top performance and to avoid severe tumbling accidents.

At a given hill (Fig. 1), the jump length depends on the inrun velocity (v_0), and the take-off velocity perpendicular to the ramp (v_{p0}) being the initial conditions, as well as the forces that act during the flight on the athlete and his equipment, i.e. the gravitational force (F_g) and the aerodynamic forces drag (F_d) and lift (F_l)[2]. Considering these forces, Straumann was the first to publish the equations of motion of a ski jumper (1927[2]), which provide a basis for analyzing and optimizing ski jumping. They can be solved for a given set of initial conditions and wind tunnel data as the aerodynamic forces are functions of the flight position and equipment (Fig. 1).

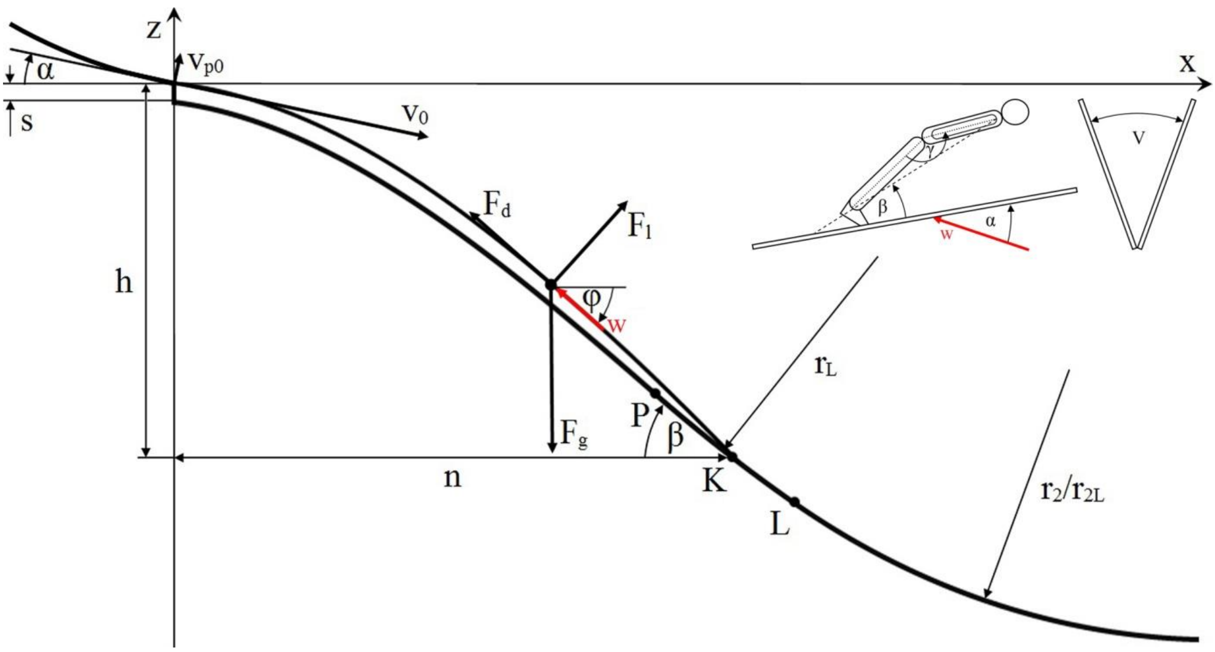

Figure 1: Flight path of a ski jumper on a hill profile. φ denotes the flight path angle which is also the angle of the relative wind vector **w** in calm wind condition. Hill profiles are modeled piecewise and the corresponding hill parameters can be found in the FIS Certificates of Jumping hills. The flight position of a ski jumper is characterized by the angle of attack of the skis α with respect to **w**, the body-ski angle β, the hip angle γ and the V-angle of the skies to each other.

Maximum jump length can be achieved by maximizing the inrun velocity and the take-off velocity perpendicular to the ramp and by optimizing the aerodynamic forces during the flight[3]. The inrun velocity can be maximized by using a ski wax that minimizes friction between the skis and the track and by minimizing the drag force, which can be achieved by adopting a crouched position. The last part of the inrun is used for the take-off. Here, the athlete has to maximize the take-off velocity perpendicular to the ramp and to create an optimum forward rotating angular momentum within only about 0.3 s[3-7]. The take-off velocity perpendicular to the ramp is maximized by a powerful and coordinated extension of the body. However, the body extension increases both drag and lift[6], which increases the take-off velocity perpendicular to the ramp but has negative consequences for the inrun velocity and the initial flight[6,7]. In parallel, the athlete must shift the center of gravity ahead of the ground reaction force vector (opposite to the take-off force vector) in order to create a forward rotating angular momentum, which is of utmost importance for achieving aerodynamically optimal and stable flight positions[3-5,7,8]. The difficult optimization task during take-off is constrained by the athlete´s features and abilities and Vaverka et al.[9] and Virmavirta et al.[7] have found distinctively different take-off techniques in elite athletes that may solve the task equally successful.

The optimization task during the flight is to find flight positions within an admissible range that optimize aerodynamic forces. The range of admissible flight positions is constrained by the athlete´s features and abilities and the pitching moment, which needs to be controlled so that the flight remains stable[3]. Remizov was the first to examine the optimal flight style (1984[10]) using Pontryagin´s minimum principle[11]. Based on wind tunnel measurements of *parallel-style* flight positions, he computed the optimum angle of attack (of the body) in the second half of the flight phase. He showed that the optimal time course of the angle of attack depends on the initial flight speed and increases in a convex way up to 45°.

Since the *V-style* was introduced by Jan Boklöv in 1985, the aerodynamic forces acting in the flight phase have become the predominant performance factors in ski jumping[3]. The biomechanics and aerodynamics of modern ski jumping was analysed by various field studies[4-9,12-15] and computer simulations[12-15]. Müller et al. (1995[12], 1996[13]) and Schmölzer and Müller (2002[14], 2005[15]) were the first to use time functions of the drag and lift area based on field studies and wind tunnel measurements for realistic computer simulations of *V-style* ski jumping. The studies suggest that the angle of attack α of the skis should be small right after the take-off but should then gradually increase during the flight[7,14], as already found by Remizov[10]. The body-ski angle β should decrease as fast as possible to a minimum value[4,7,8,14], and the hip angle γ and V-angle should increase directly after the take-off to approximately 160°, and 35°, respectively[14].

In 2004, Seo et al.[16] published an optimization study on the basis of wind tunnel measurements of *V-style* flight positions with respect to β and the V-angle. They also considered the pitching moment and kept one control constant at each simulation. However, they computed flight position time courses which cannot be found in field studies[7,13,14,15]. We have developed an optimization algorithm that takes into account both the angle of attack α of the skis and the body-ski angle. The algorithm is based on Pontryagin´s minimum principle and a penalty function[17] derived from flight position constraints. We showed that α should increases during the flight, whereas β should be as low as possible in order to maximize jump length with respect to flight stability[18]. However, as distinguished from previous assumptions, it was shown that β should increase again in the last part of the flight. In addition, we found by varying the constraints that there are various possible time courses of α and β to reach comparable jump lengths (Fig. 2)[18]. A small decrease of jump length due to a different flight style can easily be compensated by one of the other performance factors[3,12,13,14]. Even small changes in external wind can easily mask the difference[13,14]. In consequence, individual athletes can develop their individual optimum which is to be tuned with their personal features and abilities[18]. This corresponds to two field studies during the 2002 Winter Olympic Games, which illustrated that the medallists used distinctively different flight styles[7,15]. The impact of aerodynamic forces on jump length strongly increases on ski flying hills compared to large and normal hills[13,18]. For this reason, the optimum flight technique on ski flying differs from the optimum flight style on normal and large hills[18].

Elite athletes differ substantially in their take-off velocities perpendicular to the ramp. A range of v_{p0} = 2-3 ms^{-1} can be observed depending on their individual technique and muscle force[4,6,19]. In this study, the effect of the take-off velocities perpendicular to the ramp on the flight style is examined using the presented optimization algorithm. For this purpose, the hill profiles of Esto-Sadok, Russia (HS 106 m and HS 140 m, host of the Olympic Winter Games 2014) and Harrachov, Czech Republic (HS 205 m, host of the Ski Flying World Championships 2014) were used.

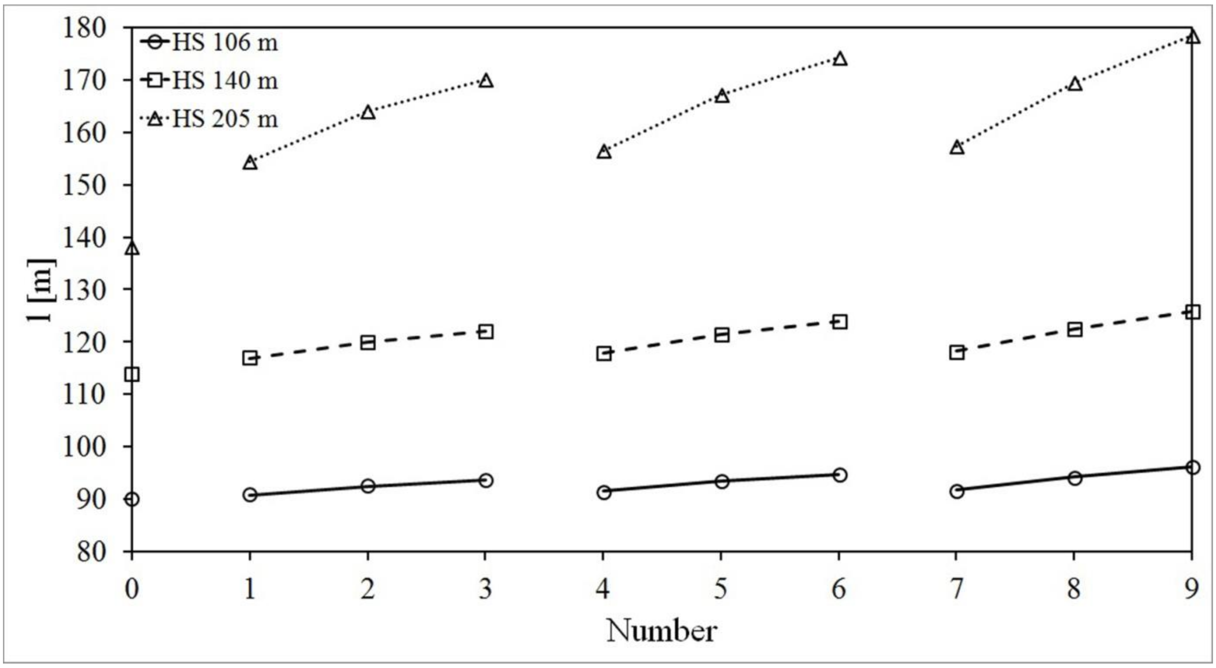

Figure 2: Jump lengths resulting from optimized flight styles with various constraints for all three hill sizes[18]. Study number 0 denotes the reference jump A, developed by Schmölzer and Müller by means of field studies[14]. α was limited to 35° (1,2,3), 40° (4,5,6) and 45° (7,8,9), respectively. β was limited with respect to the reference jump A, at least to 0°. The difference between $\beta_{ref\ A}(t)$ and $\beta_{min}(t)$ was chosen to be $\Delta\beta_{ref\ A} = 0°$ (1,4,7), –5° (2,5,8) and –10° (3,6,9), respectively. Please note, that the stated constraints in[18] are not correct. The hills of Esto-Sadok HS 106 m, HS 140 m and Harrachov HS 205 m were used.

2 OPTIMIZATION ALGORITHM

2.1 Equations of motion

The initial conditions of the flight path are the inrun velocity (v_0) and take-off velocity perpendicular to the ramp (v_{p0}). During the flight, the gravitational force (F_g), and the aerodynamic forces drag (F_d) and lift (F_l) act on the athlete with his equipment and determine the flight path:

$$F_g = mg, \tag{1}$$

$$F_d = \frac{\rho}{2} D w^2, \tag{2}$$

$$F_l = \frac{\rho}{2} L w^2, \tag{3}$$

with the relative wind vector $\mathbf{w}$ being the sum of the external wind $\mathbf{v_w}$ and the velocity $\mathbf{v}$ of the ski jumper:

$$\mathbf{w} = \mathbf{v_w} - \mathbf{v}. \tag{4}$$

The air density is a function of the air pressure with

$$\rho = \frac{p}{RT}, \tag{5}$$

where T is the absolute temperature and $R = 288.3$ JK^{-1}kg^{-1} the gas constant. In this study, the air density is chosen to be $\rho = 1.15$ kgm^{-3} according to the ICAO norm-atmosphere at 650 m sea level (Esto-Sadok: 600 m, Harrachov: 700 m). The mass of the athlete and the equipment is set to $m = 72$ kg and the gravitational acceleration is $g = 9.81$ ms^{-2}.

$D = c_d A$ and $L = c_l A$ are the drag and lift areas, which can be measured in a wind tunnel for any flight position. Wind tunnel measurements corresponding to the range of flight positions from take-off until $t = 0.7$ s are still missing. Tabulated functions $D = D(t)$ and $L = L(t)$ developed by Schmölzer and Müller (reference jump A[14]) are used in this time span. Flight style optimization begins at $t = 0.7$ s and we use $\mathbf{u}(t) = (\alpha(t), \beta(t))^T$ as controls. The data for the drag and lift areas is provide by bicubic polynomials $D(\mathbf{u})$ and $L(\mathbf{u})$ that are fitted based on a comprehensive set of wind tunnel data[14,20]. The range of measurement was α: 20-45° and β: 0-20°, and the hip angle γ and the V-angle were held constant at the aerodynamically advantageous angles 160° and 35°, respectively[14,20].

A system of four coupled nonlinear differential equations describes the motion of a ski jumper during the flight phase and provides the basis for solving the constrained optimization problem. Transformed to a first order system, the differential equations of motion without consideration of external wind read

$$\dot{\mathbf{s}} = \begin{bmatrix} \dot{v}_x \\ \dot{v}_z \\ \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \frac{\rho\sqrt{v_x^2+v_z^2}}{2m}(D(\mathbf{u})v_x + L(\mathbf{u})v_z) \\ \frac{\rho\sqrt{v_x^2+v_z^2}}{2m}(D(\mathbf{u})v_z - L(\mathbf{u})v_x) - g \\ v_x \\ v_z \end{bmatrix}, \tag{6}$$

with $\mathbf{s}(t)$ being the state matrix and the initial conditions

$$\mathbf{s}(0) = (v_x(0), v_z(0), 0, 0)^T, \tag{7}$$

with

$$\mathbf{v}(0) = v_0 + v_{p0}. \tag{8}$$

Inrun velocities used in this study for Esto-Sadok HS 106 m, HS 140 m and Harrachov HS 205 m are $v_0 = 25.0$ ms^{-1}, 26.5 ms^{-1} and 28.5 ms^{-1}, respectively according to the FIS Certificates of jumping hills. At the flight time T, the ski jumper´s flight path intersects the hill profile $P_h: z(x)$ and the height above ground is $h\big(x(T), z(T)\big) = 0$ m. This is considered to be the landing point and changing flight positions during the landing preparation are not taken into account.

2.2 Formulation of the optimization problem

Besides maximizing the jump length l (equivalent to the horizontal distance $x(T)$), the athlete has to balance the pitching moment, with respect to his features and abilities. Therefore, the individual range of the controls α and β is limited to α_{max} and $\beta_{min}(t)$. In order to avoid unrealistic position changes of β, the range of admissible β-values is chosen with respect to the reference jump A[14]:

$$\beta_{min}(t) = \beta_{ref\,A}(t) - \Delta\beta_{ref\,A}. \tag{9}$$

The following optimization study is performed with respect to an only just stable flight meaning that $\alpha_{max} = 45°$ and $\Delta\beta_{ref\,A} = -10°$, while $\beta_{min}(t)$ must not go below 0° according to wind tunnel measurements and field studies[13,21].

The constrained optimization problem with free final state and time starting at $t_0 = 0.7$ s has the following form:
Minimize

$$J = \phi\big(\mathbf{s}(T)\big) = -x(T) \tag{10}$$

subject to the system of first order dynamic constraints $\dot{\mathbf{s}}(\mathbf{s}(t), \mathbf{u}(t))$ with its initial conditions

$$\mathbf{s}(t_0) = (v_x(t_0)\ v_z(t_0)\ x(t_0)\ z(t_0))^T \tag{11}$$

and terminal condition at the landing point

$$h\big(x(T), z(T)\big) = 0 \text{ m}. \tag{12}$$

The constraints on the controls are

$$\mathbf{u}(t) \in \{(\alpha(t), \beta(t))^T | \alpha(t) \le a_{max}, \beta(t) \ge \beta_{min}(t)\}. \tag{13}$$

2.3 Solving of the optimization problem

For solving the optimization problem, a Hamiltonian is constructed:

$$\mathcal{H}(\mathbf{s}(t), \mathbf{u}(t), \boldsymbol{\lambda}(t)) = \boldsymbol{\lambda}^T(t) \cdot \mathbf{f}\big(\mathbf{s}(t), \mathbf{u}(t)\big), \tag{14}$$

with the adjoint variables $\boldsymbol{\lambda}$. When applying Pontryagin´s minimum principle[10], the necessary conditions for $\mathbf{u}^*(t)$ to be optimal are:

$$\dot{\mathbf{s}}^*(t) = \left.\frac{\partial \mathcal{H}\big(\mathbf{s}^*(t),\ \mathbf{u}^*(t),\ \boldsymbol{\lambda}(t)\big)}{\partial \boldsymbol{\lambda}}\right|_{\boldsymbol{\lambda}=\boldsymbol{\lambda}^*} = f\big(\mathbf{s}^*(t), \mathbf{u}^*(t)\big), \tag{15}$$

$$\dot{\boldsymbol{\lambda}}^*(t) = -\left.\frac{\partial \mathcal{H}\big(\mathbf{s}(t), \mathbf{u}^*(t), \boldsymbol{\lambda}^*(t)\big)}{\partial \mathbf{s}}\right|_{\mathbf{s}=\mathbf{s}^*} \tag{16}$$

and

$$\mathcal{H}(\mathbf{s}^*(t), \mathbf{u}^*(t), \boldsymbol{\lambda}^*(t)) = \min_{\mathbf{u}} \mathcal{H}(\mathbf{s}^*(t), \mathbf{u}(t), \boldsymbol{\lambda}^*(t)), \tag{17}$$

for all admissible controls, all $t \in (t_0, T^*)$, and the transversality condition

$$\boldsymbol{\lambda}^*(T^*) = \left.\left(\frac{\partial \phi}{\partial \mathbf{s}} + \frac{\partial h}{\partial \mathbf{s}} \cdot \upsilon\right)\right|_{T^*}, \tag{18}$$

with υ that can be obtained by the condition at the optimal final time T^*:

$$\mathcal{H}\big(\mathbf{s}^*(T^*), \mathbf{u}^*(T^*), \lambda^*(T^*)\big) = 0. \tag{19}$$

The constraints can be enforced by a penalty function[17]:

$$P(\mathbf{u}(t)) = (max\{0, \alpha(t) - a_{max}\})^2 + (max\{0, \beta_{min}(t) - \beta(t)\})^2. \tag{20}$$

Thus, the optimal controls $\mathbf{u}^*(t)$ can be found from an unconstrained problem by minimizing the convex function

$$q(c, \mathbf{s}^*(t), \mathbf{u}(t), \boldsymbol{\lambda}^*(t)) = \mathcal{H}(\mathbf{s}^*(t), \mathbf{u}(t), \boldsymbol{\lambda}^*(t)) + cP(\mathbf{u}(t)), \tag{21}$$

using the penalty parameter $c = 10^{10}$. The optimal controls $\mathbf{u}^*(t)$ are global maxima as well as the associated jump lengths l.

The optimization algorithm is divided into the parts *simulation* and *optimization*:
Given are the initial conditions, eq. (7), (8) and an initial guess for the controls $\mathbf{u}^0(t)$.
Repeat

Simulation:

1. Solving the equations of motion (6) forward in time with Heun´s method and $\Delta t = 10^{-4}$ s by using reference jump A up to t_0 until the terminal conditions, eq. (12), are satisfied.

Optimization:

2. Solving eq. (16) backwards in time with (18) also with Heun´s method and $\Delta t = 10^{-4}$ s.
3. Minimize eq. (21) for $t \in (t_0, T^*)$ with Newton´s method and a tolerance of $\varepsilon = 10^{-4}$ in order to get the new controls which are used in step 1 again.

Until J has converged to the minimum value with a tolerance of $\varepsilon = 10^{-4}$ m and the corresponding controls $\mathbf{u}^*(t)$ are found.

3 RESULTS AND DISCUSSION

Fig. 3 shows flight path simulations of optimized flight styles with respect to a range of take-off velocities perpendicular to the ramp of $v_{p0} = 2\text{-}3$ ms^{-1} observed in elite ski jumping[4,19]. The hill profile of the HS 205 m ski flying hill in Harrachov is used. The corresponding jump lengths are 166.4 m ($v_{p0} = 2.0$ ms^{-1}), 178.4 m ($v_{p0} = 2.5$ ms^{-1}) and 188.3 m ($v_{p0} = 3.0$ ms^{-1}), respectively.

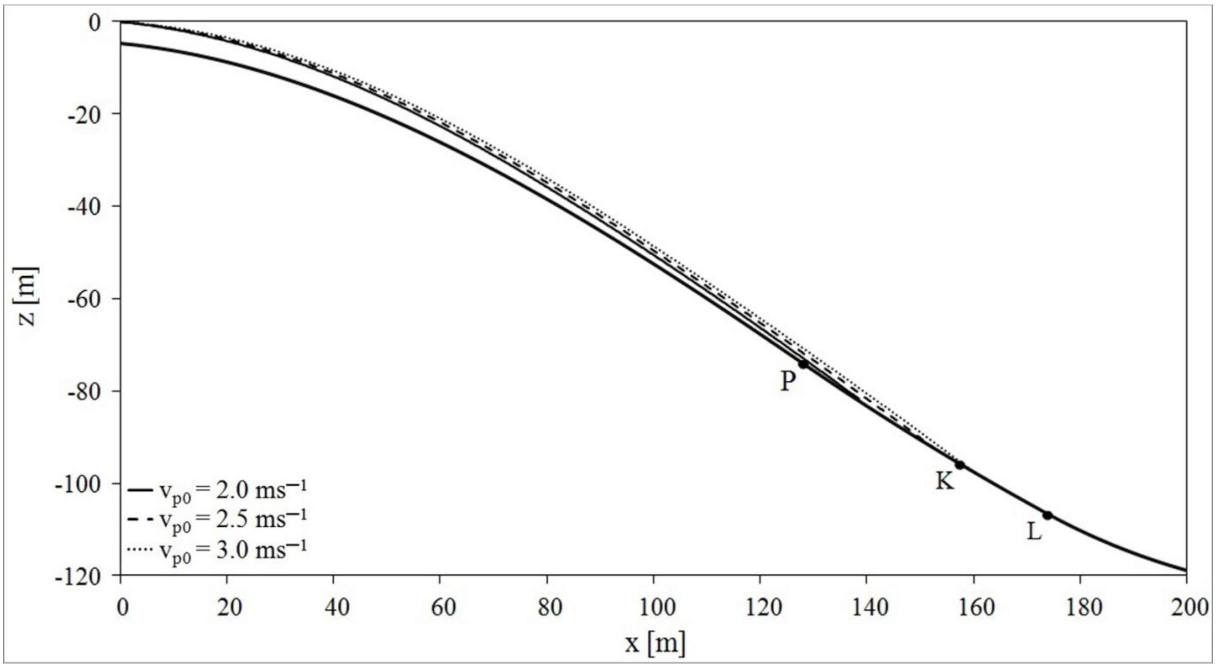

Figure 3: Flight paths of flight styles optimized for various v_{p0} on the HS 205 m ski flying hill. P-Point: 148 m, K-Point: 185 m, L-Point (HS): 205 m.

Fig. 4 shows the optimized time courses for the angle of attack α and the body-ski angle β. The optimal angles of attack $\alpha^*(t)$ of the skis decreases slightly with increasing v_{p0}. This is also found on the normal and large hill and corresponds to the optimization studies of Remizov[8]. Since jump length and thus flight time is increased at higher v_{p0}, the increase of $\beta^*(t)$ in the last part of the flight occurs later. Although jump lengths differ substantially, the flight style differences at each v_{p0} are small. This is also found on the normal and large hill.

In the next step, the length effect of flight style optimization with respect to v_{p0} is analysed. For this, the flight style optimized for $v_{p0} = 2.5$ ms^{-1} is applied to ski jumps with $v_{p0} = 2.0$ ms^{-1} and $v_{p0} = 3.0$ ms^{-1}. The resulting jump lengths are then compared to the jump lengths obtained with flight styles optimized for each v_{p0}. The maximal difference is 1.3 m in the case of $v_{p0} = 2.0$ ms^{-1} on the ski flying hill. Thus, it can be concluded that v_{p0} has only minor effect on flight style optimization in elite ski jumping and $v_{p0} = 2.5$ ms^{-1} can be used as a representative value for further optimization studies.

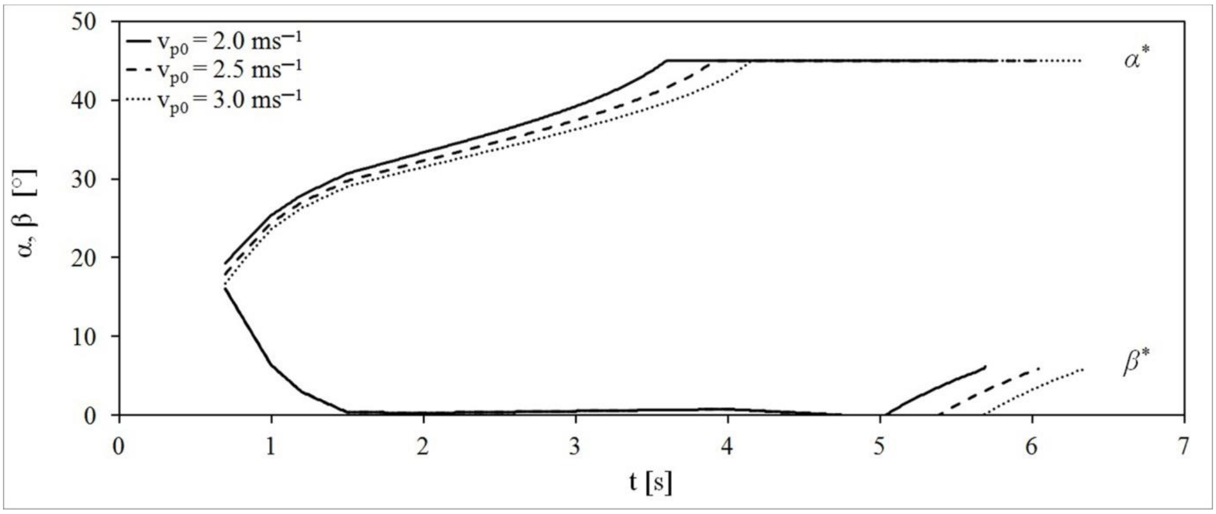

Figure 4: Optimized time courses $\alpha^*(t)$ and $\beta^*(t)$ for various v_{p0} on the HS 205 m ski flying hill.

4 OUTLINE FOR FUTURE WORK

Optimization studies can be used advantageously for guiding the training in elite ski jumping. Since the equipment has changed in the past years and aerodynamic data of the initial flight is still lacking, new field studies in combination with detailed wind tunnel measurements of athletes using the latest equipment are necessary for further research. This data would provide improved arguments for constraining flight positions and enable a better understanding of individual flight style optimization.

Pontryagin´s minimum principle is not only applicable in ski jumping but also in other sports that can be described by ordinary differential equations. Using computer simulations in combination with an optimization algorithm provides a useful basis for improving sports performance and equipment.

REFERENCES

[1] Gasser, H.H. *Standards for the construction of jumping hills - 2008*. International Ski Federation (2008).

[2] Straumann, R. *Vom Skiweitsprung und seiner Mechanik*. Jahrbuch des Schweizerischen Ski Verbandes, Selbstverlag des SSV (1927).

[3] Müller, W. Determinants of Ski-Jump performance and implications for health, safety and fairness. *Sports Med.* (2009) **39**(2):85-106.

[4] Schwameder, H. and Müller, E. Biomechanische Beschreibung und Analyse der V-Technik im Skispringen. *Spectr. Sportwiss.* (1995) **7**(1): 5-36.

[5] Virmvirta M., Isolehto, J. and Komi, P. Take-off analysis of the Olympic ski jumping competition (HS-106 m). *J. Biomech.* (2009) **42**(8): 1095-1101.

[6] Virmavirta, M., Kivekäs, J., Komi, P. Take-off aerodynamics in ski jumping. *J. Biomech.* (2001) **34**(4): 465-470.

[7] Virmavirta, M., Isolehto, J., Komi, P., Brüggemann, G.P., Müller, E. and Schwameder, H. Characteristics of the early flight phase in the Olympic ski jumping competition. *J. Biomech.* (2005) **38**(11): 2157-2163.

[8] Arndt, A., Brüggemann, G.P., Virmavirta, M. and Komi, P. Techniques used by Olympic ski jumpers in the transition from takeoff to early flight. *J. Appl. Biomech.* (1995) **11**: 224-237.

[9] Vaverka, F., Janura, M., Elfmark, M., Salinger, J. and McPherson, M. Inter- and intra-individual variability of the ski-jumper´s take-off. *Science and Skiing*, E&FN Spon (1997): 61-71.

[10] Remizov, L.P. Biomechanics of optimal flight in ski jumping. *J. Biomech.* (1984) **17**(3): 167-171.

[11] Pontryagin, L.S., Boltyanskii, V.G., Gamkrelidze, R.V. and Mishchenko, E.F. *The mathematical theory of optimal processes*, Wiley Interscience (1962).

[12] Müller, W., Platzer, D. and Schmölzer, B. Scientific approach to ski safety. *Nature* (1995) **375**: 455.

[13] Müller, W., Platzer, D. and Schmölzer, B. Dynamics of human flight on skis: improvements on safety and fairness in ski jumping. *J. Biomech.* (1996) **29**(8): 1061-1068.

[14] Schmölzer, B. and Müller, W. The importance of being light: Aerodynamic forces and weight in ski jumping. *J. Biomech.* (2002) **35**(8): 1059-1069.

[15] Schmölzer, B. and Müller, W. Individual flight styles in ski jumping: results obtained during Olympic Games competitions. *J. Biomech.* (2005) **38**(5): 1055-1065.

[16] Seo, K., Murakami, M. and Yoshida, K. Optimal flight technique for V-style ski jumping. *Sports Engineer.* (2004) **7**(1): 97-104.

[17] Chong, E.K.P. and Żak, S.H. *An introduction to optimization*, 3rd edition, Wiley Interscience (2008).

[18] Jung, A., Staat, M. and Müller, W. Flight style optimization in ski jumping on normal, large, and ski flying hills. *J. Biomech.* (2014) **47**(3): 716-722.

[19] Müller, W. Performance factors in ski jumping. *Sport aerodynamics*, CISM Courses and Lectures, vol. 506, Springer (2008): 139-160.

[20] Meile, W., Reisenberger, E., Mayer, M., Schmölzer, B., Müller, W. and Brenn, G. Aerodynamics of ski jumping: experiments and CFD simulations. *Exp. Fluids* (2006) **41**: 949-964.
[21] Reisenberger, E, Meile, W., Brenn, G. and Müller, W. Aerodynamic behaviour of prismatic bodies with sharp and rounded edges. *Exp. Fluids* (2004) **37**: 547-558.

Remark

This paper has been revised after the conference.