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# OPTIMIZATION OF THE FLIGHT STYLE IN SKI JUMPING

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**Abstract.** During the flight phase the athlete has to optimize the aerodynamic forces in order to maximize the jump length while keeping the flight stable, both with respect to his features and abilities. A system of first order nonlinear differential equations describes the motion of a ski jumper and provides the basis for solving this constrained optimization problem by means of an optimization algorithm and comprehensive wind tunnel measurements. An optimization algorithm was developed on the basis of Pontryagin´s minimum principle combined with a penalty function derived from flight position constraints. By varying the constraints, it has been recently shown that there are various possibilities to reach comparable jump lengths and individual athletes have to develop their individual optimum which is to be tuned with their personal features and abilities. In this study, the effect of the take-off velocity perpendicular to the ramp ($v_{p0}$) on the flight style is examined in order to deepen the understanding of the individual flight style optimization. It is shown that a reference value of $v_{p0} = 2.5$ ms$^{-1}$ can be used for optimization studies in elite ski jumping. Optimization studies can be used advantageously for guiding the individual training. Since the optimization algorithm was developed for the flight phase starting at 0.7 s, future work should be based on the consideration of the initial flight. For this purpose new comprehensive wind tunnel measurements with athletes using the latest equipment and field studies are necessary. The presented optimization approach can be applied to any sports which can be described by ordinary differential equations. That provides a useful basis for improving sports performance and equipment.

## 1 INTRODUCTION

Ski jumping is an Olympic discipline since the first Winter Games in 1924. International competitions are held on three types of hills differing in their hill size (HS), defined as the length between the edge of the take-off ramp and the end of the landing area: normal hill (85-109 m), large hill (≥109 m) and ski flying hill (>185 m)[1]. Including the four interrelated phases inrun, take-off, flight and landing ski jumping is technically very demanding since the athlete has to solve difficult optimization tasks within fractions of a second to reach top performance and to avoid severe tumbling accidents.

At a given hill (Fig. 1) the jump length depends on the inrun velocity ($v_0$) and the take-off velocity perpendicular to the ramp ($v_{p0}$) being the initial conditions as well as the forces that act during the flight on the athlete and his equipment, i.e. the gravitational force ($F_g$) and the aerodynamic forces drag ($F_d$) and lift ($F_l$)[2]. Considering these forces Straumann was the first to publish the equations of motion of a ski jumper (1927[2]) which provide a basis for analyzing and optimizing ski jumping. They can be solved for a given set of initial conditions and wind tunnel data because the aerodynamic forces are functions of the equipment and the flight position (Fig. 1).

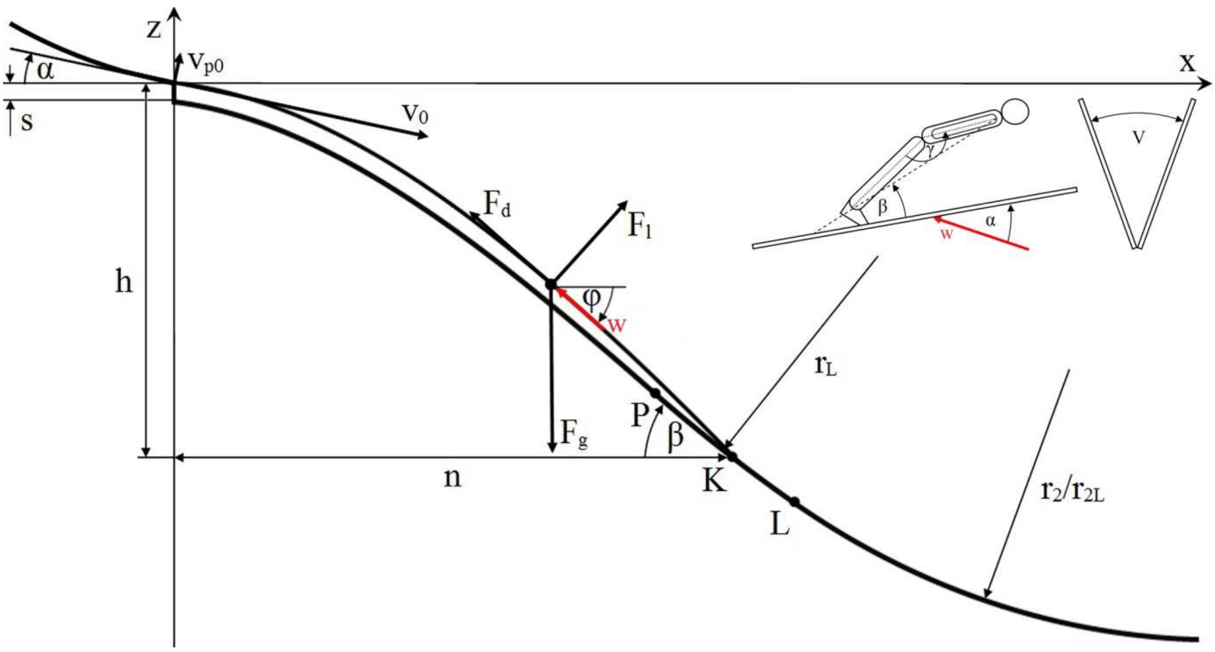


**Figure 1**: Flight path of a ski jumper on a hill profile. $\varphi$ denotes the flight path angle which is also the angle of the relative wind vector **w** in calm wind condition. Hill profiles are modeled piecewise and the corresponding hill parameters can be found in the FIS Certificates of Jumping hills. The flight position of a ski jumper is characterized by the angle of attack of the skis $\alpha$ with respect to **w**, the body-ski angle $\beta$, the hip angle $\gamma$ and the $V$-angle of the skies to each other.

In order to maximize the jump length the in-run velocity and the take-off velocity perpendicular to the ramp (initial conditions) are to be maximized, whereas the aerodynamic forces are to be optimized during the flight[3].

During the inrun phase the athlete has to maximize the inrun velocity on the one hand by using a ski waxing enabling minimum friction between the skis and the track and on the other hand by minimizing the drag force acting on him which he achieved by taking a crouched position. On the take-off ramp the take-off has to be performed subsequently within approximately 0.3 s and must not be finished either early or late as this would result in a bad performance[3]. It is the most crucial phase for ski jumping performance since it determine the initial conditions for the flight[3,4,5]. During the given time span the athlete has to maximize the take-off velocity perpendicular to the ramp by exerting a maximum take-off force due to the extension of the body. This increases the drag as well as the lift force which affect the inrun velocity negatively and the velocity perpendicular to the ramp positively [6]. Simultaneously the athlete has to produce an optimal forward rotating angular momentum around his center of gravity in order to transition into an aerodynamically optimal and stable flight position as soon as possible[3,4,5]. Therefore the ground reaction force (take-off force) vector acts behind the center of gravity in order to create a moment arm. On condition of an optimal angular momentum and the optimal flight style the athlete has to find a technique which affect both the inrun velocity and take-off velocity perpendicular to the ramp so that the jump length can be maximized. This must be done with respect to his features and abilities. Vaverka et al. [7] and Virmavirta et al. [5] have observed distinctively different techniques used by elite athletes which may solve the optimization task equally.

Since the optimization of the take-off requires the knowledge about the optimal flight style the optimization tasks are strongly connected to each other. As already mentioned the aerodynamic forces have to be optimized throughout the entire flight. But this cannot be done without making sure that the flight remains stable by controlling the pitching moment which is also a function of the flight position[3]. The athlete´s features and abilities have also to be taken into account in this optimization task.

Remizov was the first to examine the optimal flight style (1984[8]) using Pontryagin´s minimum principle[9]. By means of wind tunnel measurements of *parallel-style* flight positions he developed an algorithm optimizing the angle of attack of the skis $\alpha$ in the second half of the flight phase. He has shown that $\alpha$ must increase in a convex way up to 45° during the flight depending on the initial flight velocity to obtain maximum jump length.

Since the *V-style* was introduced by Jan Boklöv in 1985, the aerodynamic forces acting in the flight phase have become predominant performance factors in ski jumping[3]. Arndt et al.[10] and Schwameder and Müller[4] were the first to examine the V-style technique in ski jumping (1995) followed by Virmavirta et al. (2005[11]) by means of statistical methods applied to field studies. Müller et al. (1995[12], 1996[13]) and Schmölzer and Müller (2002[14], 2005[15]) developed a modelling approach in order to analyze the V-style technique by means of wind tunnel measurements and previous field studies. In summary it can be stated that $\alpha$ should increase during the flight[11,14], as already found out by Remizov[8], the body-ski angle $\beta$ should decrease as fast as possible to a minimum value[4,10,11,14] and the hip angle $\gamma$, as well as the $V$-angle should increase directly after the take-off to aerodynamically advantageous angles of approximately 160° and 35°, respectively[14]. By means of the angular momentum produced during the take-off the athlete can rotate forward. In doing so he has to lift up the skis via a dorsal flexion in the ankle joint. As a result an advantageous time courses of $\alpha$ can be achieved as well as a fast decrease of $\beta$[4,10,11,13,15]. In order to stop the angular momentum at the right moment and keep the flight stable afterwards the backward rotating pitching moment must be controlled simultaneously[5,12,13,15].

Seo et al.[16] have published in 2004 an optimization study on the basis of wind tunnel measurements of a *V-style* flight positions with respect to $\beta$ and the $V$-angle as controls considering the pitching moment affecting the time course of the body angle with respect to the horizontal axis. After a flight time of 0.4 s the optimization algorithm starts with initial flight velocities and an initial angular momentum. They have kept one control constant at each study and got time courses which cannot nearly be found in both the optimization study before and the field studies which were performed so far[11,13,14,15].

We developed an optimization algorithm taking into account both the angle of attack and the body-ski angle starting at 0.7 s on the basis of Pontryagin´s minimum principle and a penalty function[17] derived from flight position constraints. As already reported, optimization studies showed clearly that $\alpha$ should increase, whereas $\beta$ should decrease in order to maximize jump length with respect to flight stability[18]. However by varying the constraints it has been shown that there are various possibilities to reach comparable jump lengths (Fig. 2)[18]. A small decrease of jump length due to a different flight style can easily be compensated by one of the other performance factors[3,12,13,14]. Even small changes in external wind can easily mask the difference[13,14]. Consequently individual athletes have to develop their individual optimum which is to be tuned with their personal features and abilities[18]. That corresponds to two field studies during the 2002 Winter Olympic Games which illustrated that the medallists used distinctively different flight styles[11,15]. The impact of aerodynamic forces on jump length strongly increases on ski flying hills compared to large and normal hills[13,18]. For this reason, athletes have to adjust their flight style in ski flying competitions[18]. As distinguished from the previous results the optimization studies have shown that $\beta$ should increase again in the last part of the flight. However this only results in significantly farther jumps on the ski flying hill in the case of $\alpha < 40°$ and $\beta$ minimized to $<5°$ before[18].

Elite athletes differ substantially in their take-off velocities perpendicular to the ramp. A range of $v_{p0} =$ 2-3 ms$^{-1}$ can be observed depending on their individual technique and muscle force[4,6,19]. In this study the effect of the take-off velocities perpendicular to the ramp on the flight style is examined using the presented optimization algorithm to get a detailed understanding of individual flight style optimization. For this purpose the hill profiles of Esto-Sadok, Russia (HS 106 m and HS 140 m, host of the Olympic Winter Games 2014) and Harrachov, Czech Republic (HS 205 m, host of the Ski Flying World Championships 2014) were used.

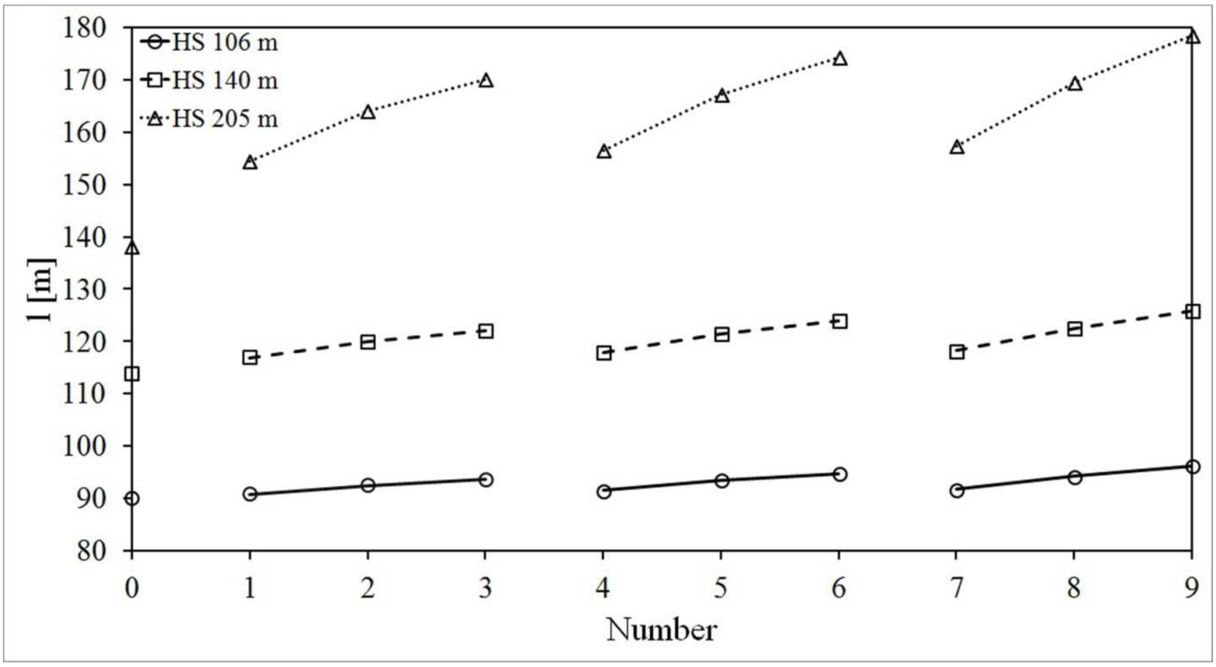


**Figure 2**: Jump lengths resulting from optimized flight styles with various constraints for all three hill sizes[18]. Study number 0 denotes the reference jump A, developed by Schmölzer and Müller by means of field studies[14]. $\alpha$ was limited to 35° (1,2,3), 40° (4,5,6) and 45° (7,8,9), respectively. $\beta$ was limited with respect to the reference jump A, at least to 0°. The difference between $\beta_{ref\ A}(t)$ and $\beta_{min}(t)$ was chosen to be $\Delta\beta_{ref\ A} = 0°$ (1,4,7), –5° (2,5,8) and –10° (3,6,9), respectively. Please note, that the stated constraints in[18] are not correct. The hills of Esto-Sadok HS 106 m, HS 140 m and Harrachov HS 205 m were used.

## 2 OPTIMIZATION ALGORITHM

### 2.1 Equations of motion

During the flight, the gravitational force ($F_g$) and the aerodynamic forces drag ($F_d$) and lift ($F_l$) act on the athlete with his equipment and determine the flight path of the centre of gravity with the initial conditions inrun velocity ($v_0$) and take-off velocity perpendicular to the ramp ($v_{p0}$):

$$F_g = mg, \tag{1}$$

$$F_d = \frac{\rho}{2} D w^2, \tag{2}$$

$$F_l = \frac{\rho}{2} L w^2, \tag{3}$$

with the relative wind vector $\mathbf{w}$ being the sum of the external wind $\mathbf{v_w}$ and the velocity $\mathbf{v}$ of the ski jumper:

$$\mathbf{w} = \mathbf{v_w} - \mathbf{v}. \tag{4}$$

The air density is a function of the air pressure with

$$\rho = \frac{p}{RT}, \tag{5}$$

where $T$ is the absolute temperature and $R = 288.3$ JK$^{-1}$kg$^{-1}$ the gas constant. In the following study the air density was chosen to be $\rho = 1.15$ kgm$^{-3}$ according to the ICAO norm-atmosphere at 650 m sea level (Esto-Sadok: 600 m, Harrachov: 700 m). The mass of the athlete and his equipment was set to $m = 72$ kg according to the competition rules of the international ski federation (FIS) and the gravitational acceleration is $g = 9.81$ ms$^{-2}$.

$D = c_d A$ and $L = c_l A$ are the drag and lift areas which can be measured in a wind tunnel for any flight position. Wind tunnel measurements corresponding to the range of flight positions from take-off until $t = 0.7$ s are still missing. Tabulated functions $D = D(t)$ and $L = L(t)$ developed by Schmölzer and Müller on the basis of field studies and wind tunnel measurements (reference jump A[14]) were used in this time span. We are using $\mathbf{u}(t) = (\alpha(t), \beta(t))^T$ as controls. For the optimization starting subsequently bicubic polynomials $D(\mathbf{u})$ and $L(\mathbf{u})$ were fitted on the basis of a comprehensive set of wind tunnel data[14,20]. The range of measurement was for $\alpha$: 20-45° and for $\beta$: 0-20°, whereas the hip angle $\gamma$ and the $V$-angle were held constant at the aerodynamically advantageous angles 160° and 35°, respectively[14,20].

A system of four coupled nonlinear differential equations describe the motion of a ski jumper during the flight phase and provides the basis for solving the constrained optimization problem by means of an optimization algorithm and comprehensive wind tunnel measurements. Transformed to a first order system the differential equations of motion without consideration of external wind read

$$\dot{\mathbf{s}} = \begin{bmatrix} \dot{v}_x \\ \dot{v}_z \\ \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \frac{\rho\sqrt{v_x^2+v_z^2}}{2m}(D(\mathbf{u})v_x + L(\mathbf{u})v_z) \\ \frac{\rho\sqrt{v_x^2+v_z^2}}{2m}(D(\mathbf{u})v_z - L(\mathbf{u})v_x) - g \\ v_x \\ v_z \end{bmatrix}, \tag{6}$$

with $\mathbf{s}(t)$ being the state matrix and the initial conditions

$$\mathbf{s}(0) = (v_x(0), v_z(0), 0, 0)^T, \tag{7}$$

with

$$\mathbf{v}(0) = v_0 + v_{p0}. \tag{8}$$

Inrun velocities used in this study for Esto-Sadok HS 106 m, HS 140 m and Harrachov HS 205 m are $v_0 = 25.0$ ms$^{-1}$, 26.5 ms$^{-1}$ and 28.5 ms$^{-1}$, respectively according to their FIS Certificates of jumping hills.

At the flight time $T$ the ski jumper´s flight path intersects the hill profile $P_h: z(x)$ at the landing point so that the height above ground is $h\big(x(T), z(T)\big) = 0$ m.

Flight positions with respect to the landing preparation are not considered so far in the optimization algorithm.

## 2.2 Formulation of the optimization problem

Besides maximizing the jump length $l$ (equivalent to the horizontal distance $x(T)$), the athlete has to balance the pitching moment, with respect to his features and abilities. Therefore the individual range of the controls $\alpha$ and $\beta$ is limited to $\alpha_{max}$ and $\beta_{min}(t)$. In order to avoid unrealistic position changes of $\beta$, the range of admissible $\beta$-values was chosen with respect to the reference jump A[14]:

$$\beta_{min}(t) = \beta_{ref\,A}(t) - \Delta\beta_{ref\,A}. \tag{9}$$

The following optimization study is performed with respect to a only just stable flight meaning that $\alpha_{max} = 45°$ and $\Delta\beta_{ref\,A} = -10°$, while $\beta_{min}(t)$ must not go below 0° according to wind tunnel measurements and field studies[13,21].

The constrained optimization problem with free final state and time starting at $t_0 = 0.7$ s has the following form:
Minimize

$$J = \phi(\mathbf{s}(T)) = -x(T) \tag{10}$$

subject to the system of first order dynamic constraints $\dot{\mathbf{s}}(\mathbf{s}(t), \mathbf{u}(t))$ with its initial conditions

$$\mathbf{s}(t_0) = (v_x(t_0)\; v_z(t_0)\; x(t_0)\; z(t_0))^T \tag{11}$$

and terminal condition at the landing point

$$h(x(T), z(T)) = 0 \text{ m}. \tag{12}$$

The constraints on the controls are

$$\mathbf{u}(t) \in \{(\alpha(t), \beta(t))^T | \alpha(t) \leq a_{max}, \beta(t) \geq \beta_{min}(t)\}. \tag{13}$$

## 2.3 Solving of the optimization problem

For solving the optimization problem a Hamiltonian is constructed:

$$\mathcal{H}(\mathbf{s}(t), \mathbf{u}(t), \boldsymbol{\lambda}(t)) = \boldsymbol{\lambda}^T(t) \cdot \mathbf{f}(\mathbf{s}(t), \mathbf{u}(t)), \tag{14}$$

with the adjoint variables $\boldsymbol{\lambda}$. Applying Pontryagin´s minimum principle[9] the necessary conditions for $\mathbf{u}^*(t)$ to be optimal are:

$$\dot{\mathbf{s}}^*(t) = \left.\frac{\partial\mathcal{H}(\mathbf{s}^*(t),\ \mathbf{u}^*(t),\ \boldsymbol{\lambda}(t))}{\partial\boldsymbol{\lambda}}\right|_{\boldsymbol{\lambda}=\boldsymbol{\lambda}^*} = f(\mathbf{s}^*(t), \mathbf{u}^*(t)), \tag{15}$$

$$\dot{\boldsymbol{\lambda}}^*(t) = -\left.\frac{\partial\mathcal{H}(\mathbf{s}(t), \mathbf{u}^*(t), \boldsymbol{\lambda}^*(t))}{\partial\mathbf{s}}\right|_{\mathbf{s}=\mathbf{s}^*} \tag{16}$$

and

$$\mathcal{H}(\mathbf{s}^*(t), \mathbf{u}^*(t), \boldsymbol{\lambda}^*(t)) = \min_{\mathbf{u}} \mathcal{H}(\mathbf{s}^*(t), \mathbf{u}(t), \boldsymbol{\lambda}^*(t)), \tag{17}$$

for all admissible controls and for all $t \in (t_0, T^*)$ and the transversality condition

$$\boldsymbol{\lambda}^*(T^*) = \left(\frac{\partial \phi}{\partial \mathbf{s}} + \frac{\partial h}{\partial \mathbf{s}} \cdot \upsilon\right)\Big|_{T^*}, \quad (18)$$

with $\upsilon$ can be obtained by the condition at the optimal final time $T^*$:

$$\mathcal{H}\big(\mathbf{s}^*(T^*), \mathbf{u}^*(T^*), \lambda^*(T^*)\big) = 0. \quad (19)$$

The constraints can be enforced by a penalty function[17]:

$$P(\mathbf{u}(t)) = (max\{0, \alpha(t) - a_{max}\})^2 + (max\{0, \beta_{min}(t) - \beta(t)\})^2. \quad (20)$$

Hereby the optimal controls $\mathbf{u}^*(t)$ can be found from an unconstrained problem by minimizing the convex function

$$q(c, \mathbf{s}^*(t), \mathbf{u}(t), \boldsymbol{\lambda}^*(t)) = \mathcal{H}(\mathbf{s}^*(t), \mathbf{u}(t), \boldsymbol{\lambda}^*(t)) + cP(\mathbf{u}(t)) \quad (21)$$

using the penalty parameter $c = 10^{10}$. The optimal controls $\mathbf{u}^*(t)$ are global maxima as well as the associated jump lengths $l$.

The optimization algorithm is divided into the parts *simulation* and *optimization*:

**Given** are the initial conditions, eq. (7), (8) and an initial guess for the controls $\mathbf{u}^0(t)$.

**Repeat**

*Simulation*:

1. Solving the equations of motion (6) forward in time with Heun´s method and $\Delta t = 10^{-4}$ s by using reference jump A up to $t_0$ until the terminal conditions, eq. (12) are satisfied.

*Optimization:*

2. Solving eq. (16) backwards in time with (18) also with Heun´s method and $\Delta t = 10^{-4}$ s.
3. Minimize eq. (21) for $t \in (t_0, T^*)$ with Newton´s method and a tolerance of $\varepsilon = 10^{-4}$ in order to get the new controls which are used in step 1 again.

**Until** $J$ has converged to the minimum value with a tolerance of $\varepsilon = 10^{-4}$ m and the corresponding controls $\mathbf{u}^*(t)$ are found.

## 3 RESULTS AND DISCUSSION

Fig. 3 shows flight path simulations of optimized flight styles with respect to a range of take-off velocities perpendicular to the ramp of $v_{p0} = 2\text{-}3$ ms$^{-1}$ observed in elite ski jumping[4,19] using the hill profile of the HS 205 m ski flying hill in Harrachov. The corresponding jump lengths were 166.4 m ($v_{p0} = 2.0$ ms$^{-1}$), 178.4 m ($v_{p0} = 2.5$ ms$^{-1}$) and 188.3 m ($v_{p0} = 3.0$ ms$^{-1}$), respectively.

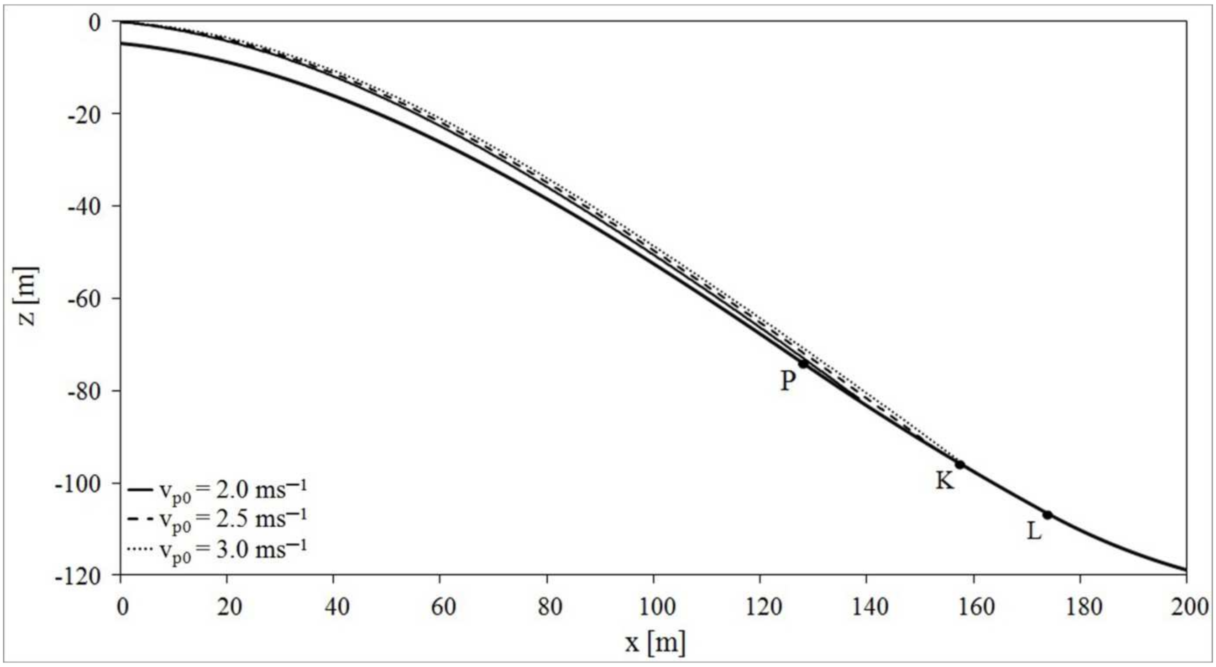


**Figure 3**: Flight paths of the optimized flight styles regarding to a range of $v_{p0}$ on the HS 205 m ski flying hill. P-Point: 148 m, K-Point: 185 m, L-Point (HS): 205 m.

Fig. 4 shows the optimized time courses for the angle of attack $\alpha$ and the body-ski angle $\beta$. With increasing $v_{p0}$ the optimal angles of attack of the skis $\alpha^*(t)$ are getting slightly smaller in the first half of the flight. By this means the drag areas are lowered and higher flight velocities can be compensated in order to keep the drag force constant. This result can also be found in the optimization studies of Remizov[8]. Due to farther jumps with higher $v_{p0}$ the increase in $\beta^*(t)$ in the last part of the flight occurs later. Although jump lengths differ substantially, the flight style differences at each $v_{p0}$ are small. This was also observed on the normal and large hill.

Thus the flight style optimized for $v_{p0} = 2.5$ ms$^{-1}$ was applied to ski jumps with $v_{p0} = 2.0$ ms$^{-1}$ and $v_{p0} = 3.0$ ms$^{-1}$. The resulting jump lengths were compared to the jump lengths obtained with flight styles optimized for each $v_{p0}$ on all three hills. Obtaining a maximal difference of 1.3 m in the case of $v_{p0} = 2.0$ ms$^{-1}$ on the ski flying hill, it can be concluded that $v_{p0} = 2.5$ ms$^{-1}$ can be used as a representative value for optimization studies in elite ski jumping.

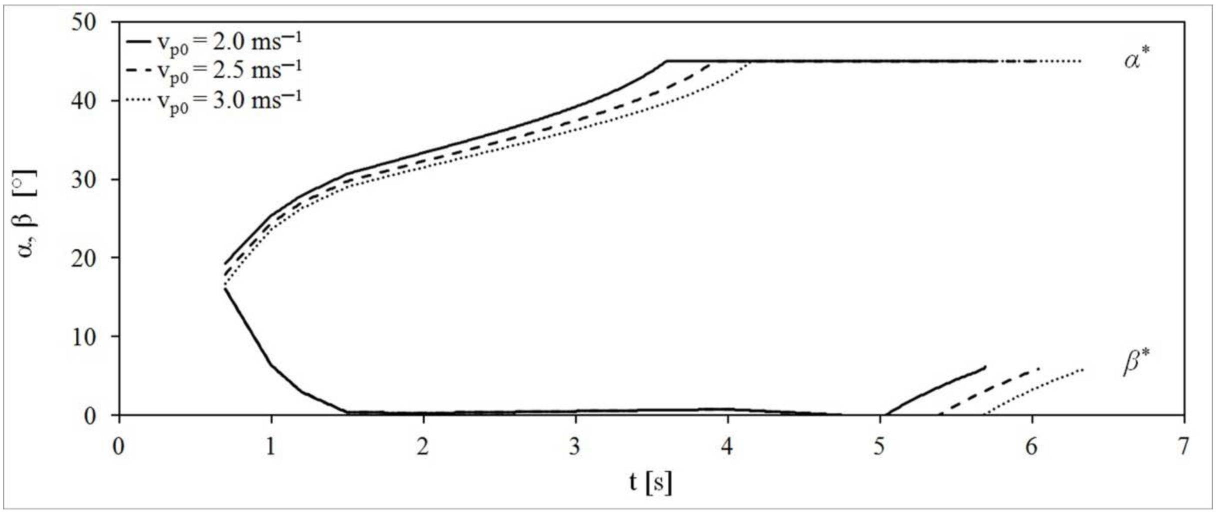


**Figure 4**: Optimized time courses $\alpha^*(t)$ and $\beta^*(t)$ regarding to a range of $v_{p0}$ on the HS 205 m ski flying hill.

## 4 OUTLINE FOR FUTURE WORK

Optimization studies can be used advantageously for guiding the training in elite ski jumping. So far, optimizations were done for the flight phase starting from $t = 0.7$ s due to missing wind tunnel measurements. After the wind tunnel measurements were performed, the equipment has changed, in particular the jumping suits are cut tighter, made of thinner material. This results in a reduction of the aerodynamic forces[20]. With a simple modelling approach it was shown that this probably has a negligible effect on the optimal time courses of $\alpha$ and $\beta$ when similar length are obtained by using higher inrun velocities[18]. However, it may affect the controlling of the flight stability. Therefore new field studies in combination with detailed wind tunnel measurements of athletes using the latest equipment have to be done including the initial flight. This would provide improved arguments for constraining flight positions and enable the extension of the optimization studies. Consequently more detailed optimization studies can be done in order to deepen the understanding of individual flight style optimization.

Pontryagin´s minimum principle is not only applicable in ski jumping but also in other sports which can be described by ordinary differential equations. Using computer simulations in combination with an optimization algorithm provides a useful basis for improving sports performance and equipment.

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**Remark**

This paper has been slightly revised after the conference.