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NONLINEAR BEHAVIOUR OF MASONRY UNDER CYCLIC LOADING

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ABSTRACT

The new trend in the seismic design philosophy is the performance based design taking the deformation of the structure into account. The capacity spectrum method, one of the displacement based design approaches, compares the seismic action with the loading capacity of the building, considering the nonlinear material behaviour with its post-peak capacity. The inelastic cyclic pushover curve of the regarded structure has to be calculated iteratively using capacity curves of single masonry shear walls as basic input. In order to facilitate the design process for the practising engineer a matrix of capacity curves depending on material combination, wall geometry and vertical loading has to be developed. Due to the large number of material, load and geometry combinations the curves have to be obtained by cyclic shear wall tests in combination with nonlinear numerical simulations and analytical solutions. A new database concept based on an intelligent combination of test data reasonable supplemented by simulation results will be introduced. The determination of single wall capacity curves using analytical solutions will be described in detail and the results will be compared with cyclic shear wall tests. Finally, the application of the database concept is demonstrated on the example of the seismic safety verification using the capacity spectrum method for typical residential buildings in Germany.

1. INTRODUCTION

Traditionally, the design of masonry buildings subject to seismic actions is carried out by means of a force-based linear approach using simplified or multimodal response spectrum modal analysis methods. The nonlinear capacity reserves of the structure are globally taken into account by simply reducing the elastic response spectrum by the behaviour factor q. This factor depends, among other things, on the ductility of the building material. For unreinforced masonry (being characterized in general as a brittle material) its value is given in European and national standards as $q = 1.5$. This low value of the behaviour factor in conjunction with the traditional force based linear elastic design makes it very difficult to comply with the structural strength requirements for masonry structures, even in regions with low to medium seismicity with long-established and time-tested (during past earthquakes) building designs.

This proves conclusively that the available capacity reserves are not being adequately taken into account by the current linear, force-based design and verification approach.

2. NONLINEAR STATIC DESIGN PROCEDURES FOR MASONRY

For ductile materials such as steel and reinforced concrete, displacement based design and verification procedures are long since being used in practice and have been implemented in design codes. Robust and generally accepted nonlinear computational models are available, with which nonlinear load-displacement relationships for buildings can be established. It is, therefore, feasible and generally accepted to design such buildings using a displacement based approach, which generally leads to a more economical solution compared to the standard force-based design method. For masonry structures, some displacement based design methods can be found in the literature, as described and commented upon in the following.

The Swiss standard SIA 2018 [20] explicitly prescribes a displacement based approach for checking the structural stability of existing masonry structures. However, it does not go into details on how to determine the force-displacement diagram of masonry structures or on how to carry out the actual verification. Instead, it recommends estimating the displacement behaviour of masonry buildings through employing force-displacement relationships for single walls from consistent standards or using test results. Clearly, applying the SIA approach in practice is no easy task, because although a general statement is made, a lot of information concerning important details is missing.

In Part 1, Eurocode 8 [7] mentions the possibility of using a displacement based verification method for masonry buildings as an alternative to the standard approach using linear-elastic analyses in conjunction with the behaviour factor q. The method relies on the determination of a target displacement based on the „Equal Displacement Approximation" and substituting the actual capacity curve of the building by an elasticideally plastic relationship. The initial stiffness is determined by equating the areas under the actual and the idealized force-displacement curve.

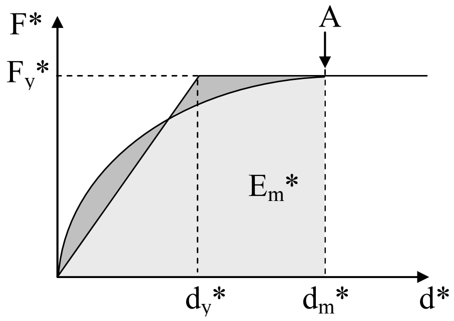

Figure 1. Idealized force-displacement curve

Based on this stiffness value, a natural period is computed and an elastic spectral displacement determined. The latter is modified by a coefficient expressing the ratio between the maximum inelastic displacement and the computed linear-elastic displacement. This coefficient has been determined by statistically evaluating the results of nonlinear time-history analyses of single degree of freedom oscillators. It happens that in the low period range ($T < 0.5$ s) this factor is very sensitive to small variations of the initial stiffness, which, it will be remembered, has been calculated only in an approximate way. Since this is exactly the range of natural periods of most masonry structures, applying the Eurocode 8 [7] procedure is not unproblematic. In addition, the method does not take the shape of the hysteretic loops into account, nor does it consider the progressive degradation of stiffness and strength. This can lead to erroneous results especially in the case of strongly pinched hysteretic loops in connection with high cyclic damage levels.

In ATC 40 [1] and in FEMA Reports 273 [12] and 274 [13] two nonlinear static methods are described which are way beyond the simplified approach of Eurocode 8 [7].The first method is the so-called displacement modification procedure (Coefficient Method). This procedure estimates the total maximum displacement of the oscillator by multiplying the elastic response, assuming initial linear properties and damping, by one or more coefficients. The coefficients are typically derived empirically from series of nonlinear response time history analyses of oscillators with varying periods and strengths. The coefficients take the stiffness and strength degradation, the shape of the hysteretic loops and also second order effects into account. The method has also been described fully in the new FEMA guideline 356 [14], and it has been further extended in FEMA 440 [15].

Another, more detailed approach is the Capacity Spectrum Method (CSM) developed by Freeman and described in ATC 40 [1]. This method is a nonlinear static method which compares the seismic action (exposure) to the loading capacity of the building, taking the nonlinear material behaviour with its post-peak capacity into account. The seismic action is represented by a response spectrum reduced due to damping. The building capacity is described by an inelastic cyclic pushover curve. Both curves are converted to acceleration-displacement response spectral diagram. The intersection point of the curves is called "Performance Point" and indicates the maximum spectral displacement for the given site spectrum (see Figure 2).

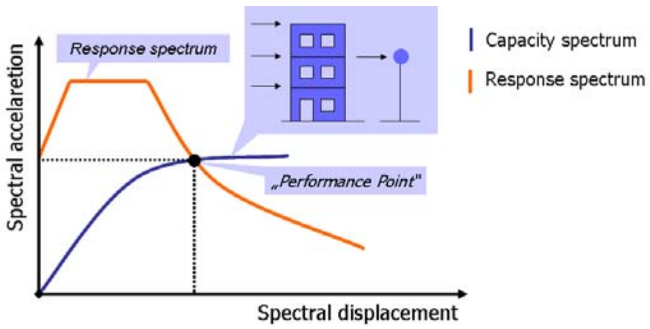

Figure 2. Capacity spectrum method and Performance Point

The application of the described displacement based approaches to masonry buildings has not been enforced in practice because the description of nonlinear wall capacity curves and structural damping as a necessary input values is difficult. Furthermore, the implementation of nonlinear push over-analysis to determine the capacity curve of the overall structure of the nonlinear behaviour of masonry with various forms of failure is very complex and needs a high computational effort. To solve this problems a performance based design on the basis of the capacity spectrum method is presented, which all requirements of the standards [4], [7] met, and on any masonry buildings can be applied.

3. DISPLACEMENT BASED DESIGN FOR MASONRY BUILDINGS

Based on the already described capacity spectrum method a new displacement based design concept has been developed taking nonlinear bearing reserves of masonry shear walls into account. Considering the masonry specific failure modes and the hysteretic damping the result are still clearly understandable and do not require the additional use of empirically determined correction coefficients. Cyclic load displacement curves (capacity curves) of single shear walls are used to determine the nonlinear push over curve of the entire structure. For determination of the so-called "Performance Point" the calculated capacity curve of the entire structure and the response spectrum representing the seismic load are transferred into a joint S_a-S_d -diagram. The following sections describe the proposed method and the determination of capacity curves by means of experimental investigations, numerical simulations and analytical approaches.

3.1 Capacity curves of single shear walls

The capacity curves of the single walls are derived by means of interpolation from a matrix consisting of capacity and damping curves at different load levels and length to height ratios (see Figure 3). These curves can be determined by experimental investigation, numerical simulation and analytical computation. This computation is a challenging task because the load displacement curve depends on the unit-mortar combination, vertical load level and wall geometry.

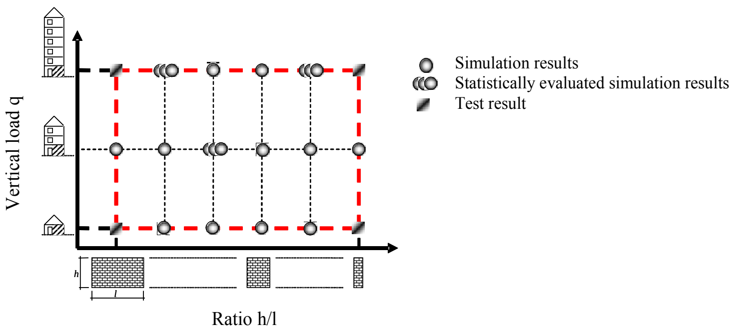

Figure 3. Matrix of capacity curves

Research on the cyclic behaviour of masonry walls has been going on for some time and quite a number of published test results can be found in the literature [2], [8], [19]. The shear bearing behaviour is currently intensively scrutinized in the ongoing EU projects ESECMaSE [6] and DISWALL [5]. The large number of experimental studies of masonry shear walls under cyclic loading has led to a deeper basic understanding of their bearing and deformation behaviour. Due to the influence of many parameters and the high costs shear wall tests depict only field trial, which can not describe the behaviour of masonry walls under cyclic load in their entirety. The unstandardised boundary conditions of the experiments make the interpretation of the test results difficult. The definition of such standards is the objective of the EU project ESECMaSE [6]. It must be observed with regard to the usability of the test results that even though numerous technical data for a wide range of combination of materials exist presently, systematic results for combination of materials under varying boundary conditions are not adequately provided.

Numerical simulations provide another opportunity to determine the shear bearing behaviour of single masonry walls. The chair of structural statics and dynamics of RWTH Aachen developed several numerical simulation models within the last years for describing the nonlinear, anisotropic and quasi-brittle behaviour of masonry. On the macro level an orthotropic hypoplastic material model based on equivalent uniaxial strain was successfully used for the simulation of unreinforced masonry shear walls [16]. Furthermore, a smeared model on the basis of multidimensional theory of plasticity with elastic plastic damage formulation had been developed incorporating the failure criteria of Mann and Mueller [17]. A push-over analysis can be simulated well with the existing models. The transfer to cyclic strain is problematic because only a few results of cyclic damage are present and no material model for stones with different perforation is currently available. At present, the prediction of the cyclic behaviour without calibration of the material parameters of the current models is not possible. Basically, the developed models will provide a handy possibility to fill the gaps in the aforementioned matrix between the experimentally determined capacity curves taking into account the influence of scattering material parameters.

Furthermore analytical approaches are also appropriate to evaluate the capacity of unreinforced masonry walls under cyclic loading [3]. The guidelines of the FEMA (Federal Emergency Management Agency) 306 [9], 307 [10], 308 [11], 356 [14] contain a detailed approach to determine load displacement curves. The idealized progress takes into account different failure modes: rocking V_r, bed joint sliding V_{bjs1}, diagonal tension V_{dt}, toe crushing V_{tc} and combined failure modes. The definition of failure mode is mainly affected by the geometrical ratio, the axial compressive force and material properties. They can be calculated according to the formulas given in Table 1.

Failure mode	Formula	
Rocking	$V_r = 0{,}9 \cdot \alpha \cdot N_{Ed} \cdot \left(\frac{l}{h_s}\right)$	N_{Ed} – axial compression force l – wall length h_s – wall height α – factor for grade of restraint
Bed joint sliding	$V_{bjs1} = \left(f_{vk0} + \mu_{sf} \cdot \sigma_{Dd}\right) \cdot A$ $V_{bjs2} = \mu_{df} \cdot A$	f_{vk0} – adhesive shear strength σ_{Dd} – compression strength A – area of mortared section μ_{sf} – static friction coefficient μ_{df} – kinetic friction coefficient
Diagonal tension	$V_{dt} = f_{dZ} \cdot A \cdot \beta \cdot \sqrt{1 + \frac{\sigma_{Dd}}{f_{dZ}}}$	f_{dZ} – diagonal tensile strength β – effect of wall geometry
Toe crushing	$V_{tc} = \alpha \cdot N_{Ed} \cdot \left(\frac{l}{h_s}\right)\left(1 - \frac{\sigma_{Dd}}{f_d}\right)$	f_d – masonry compressive strength

Table 1: Failure modes according to FEMA guidelines [9], [14]

Depending on these failure modes the shape of the capacity curve is determined through the resulting constants c, d and e as calculated in Table 2 and depicted in Figure 4.

Failure mode	V_{max}	d	e	c
Bed joint sliding	V_{bjs1}	0,004	0,008	V_{bjs2}
Rocking	V_r	$0{,}004 \cdot h_s / l$	$0{,}008 \cdot h_s / l$	$0{,}6 \cdot V_r$

Table 2: Constants depending on the failure mode according to FEMA 356 [14]

Another similar approach is given in Eurocode 8, Part 3 [7] which defines the capacity of masonry walls as minimum of the failure due to shear force or bending. This approach only takes ductile failure modes with large deformation capability into account. The deformation capability is terminated subjected to the respective failure mode by a maximum drift ratio stated in the standard and depicted below in Figure 4.

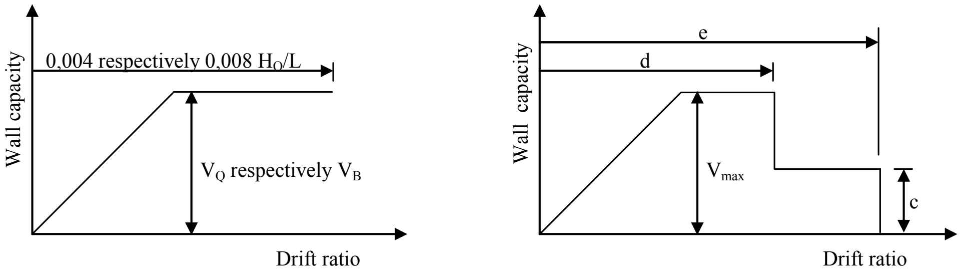

Figure 4. Idealized load displacement curve according to Eurocode 8, Part 3 (left hand) and FEMA guidelines (right hand)

The formula of FEMA contains a widespread feasible approach for determination of the wall capacity curves. It is, however, impossible to directly adapt these curves to contemporary European masonry, because the relevant deformation capacities stem from old tests using solid bricks and walls with standard mortar. Due to further development of stone-mortar combinations in the past few years the deformation capacity has changed. Thus, the formulas are not directly transferable on today's masonry and discount the degradation of stiffness due to cyclical behaviour. The simple formulas of Eurocode 8, Part 3 are based on the findings of FEMA. As a result of simplification brittle failure as well as combinations of different failure modes is not included. Both approaches do not take the stiffness reduction into account which occurs due to failure degradation.

3.2 Capacity curves of the entire structure

The capacity of the whole structure will be determined based on the capacity curves of the single shear walls. Therefore different calculation methods are available. In case of regular ground plans, unchanged shear walls from basement to top and rigid floor panels structural failure in the ground floor is generally agreed. If these assumptions are met, the capacity curve of the entire structure is related to the displacement of the ground floor. For symmetrical ground plans with symmetrical mass distribution, the capacity curve can be calculated by simple superposition of single wall-capacity curves. In case of irregular ground plans or asymmetrical mass distributions the capacity curve of the ground floor has to be calculated with a double iterative algorithm considering the rotation and translation orthogonal to the loading direction [17], [18].

This approach is sufficiently accurate for structures with few storeys. Otherwise a more detailed approach should be used taking into account the stiffness changes of all storeys. Consequently the structure is considered as a multi degree of freedom system with horizontal degrees of freedom on the floor heights and the resulting capacity curve is related to the displacement of the attic. An adaptive adjustment of the horizontal load distribution according to the current natural mode takes place including even inelastic effects and sudden changes due to failure of masonry walls [18].

3.3 Determination of the Performance Point

For determination of the so-called "Performance Point" the calculated capacity curve of the entire structure and the response spectrum representing the seismic load according to the standard are transferred into a joint S_a-S_d -diagram.

The transformation is based on the model of an equivalent single degree of freedom system and the conversion is done by using the first natural mode [17]. Therefore a multi degree of freedom system with horizontal degrees of freedom at each storey is considered (Figure 5). As described in section 3.2 failure in the ground floor is assumed. Hence stiffness degradation affected by damage is just considered in the ground floor and all other stories remain linear elastic.

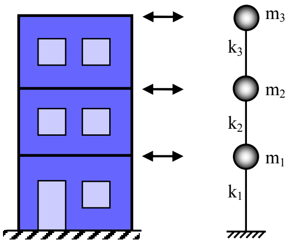

Figure 5. Equivalent system with horizontal degrees of freedom

The influence of energy dissipation within the nonlinear range of the capacity curve is considered by a reduction of the linear elastic response spectrum by means of an effective viscous damping. The effective viscous damping is the sum of the viscous damping and the equivalent viscous damping due to hysteretic behaviour. Based on local displacement curves the hysteretic damping ξ_{eq} is identified by the ratio of maximum strain energy to dissipated energy, exemplarily shown in Figure 6. With sufficient knowledge of the hysteretic damping curves of each wall the effective damping of the whole structure can be determined for each point of the load displacement curve by weighting the hysteretic damping of the shear walls.

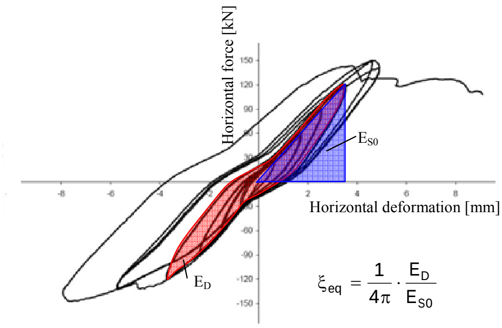

Figure 6. Calculation of the hysteretic damping

The calculation of the damped response spectrum allows a direct determination of the "Performance Point", which represents the intersection point of the capacity curve and the damped response spectrum in the S_a-S_d-diagram (see Figure 7).

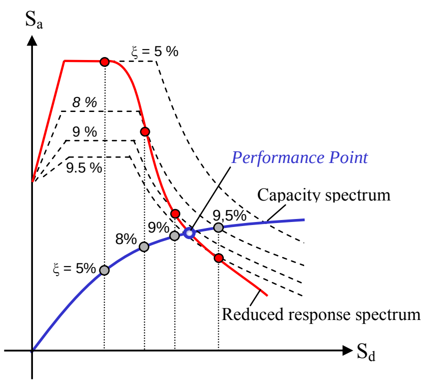

Figure 7. Determination of the "Performance Point"

4. PRACTICAL IMPLEMENTATION

Within a research project funded by the German Society for masonry (DGfM) the introduced displacement based design concept based on the capacity spectrum method was implemented into the software package M-DESIGN. All necessary input data are provided by a connected database containing the single wall capacity curves and the appropriate damping course determined from test and simulation results. Depending on the loading conditions and the length to height ratio a matrix of available data is build up for each unit mortar combination. By using a feasible interpolation algorithm the buildings capacity curves can be calculated for arbitrary wall configurations as well as geometry and loading conditions of the individual shear walls. Figure 8 shows some screenshots of the developed software and the connected database-tool as well. Overall this program provides an easy-to-use software tool for structural engineers to perform seismic safety verification of masonry buildings.

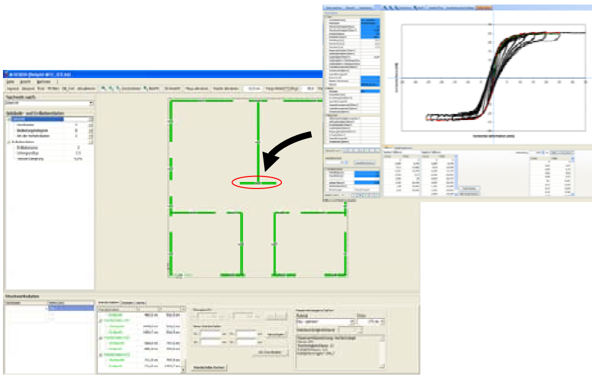

Figure 8. Software tool with connected database

5. APPLICATION EXAMPLE

In the following the displacement based design concept based on the capacity spectrum method is applied to a typical terraced house. Therefore two different databases containing experimental and analytical deduced data are used.

5.1 Design fundamentals

The first database contains the results of cyclic shear wall tests carried out within the EU project ESECMASE [6]. The determined cyclic load displacement curves of shear walls made of vertically perforated clay bricks (HLZ 12/IIa) with a length of 1.10 m and 2.20 m and a height of 2.5 m are shown in Figure 9.

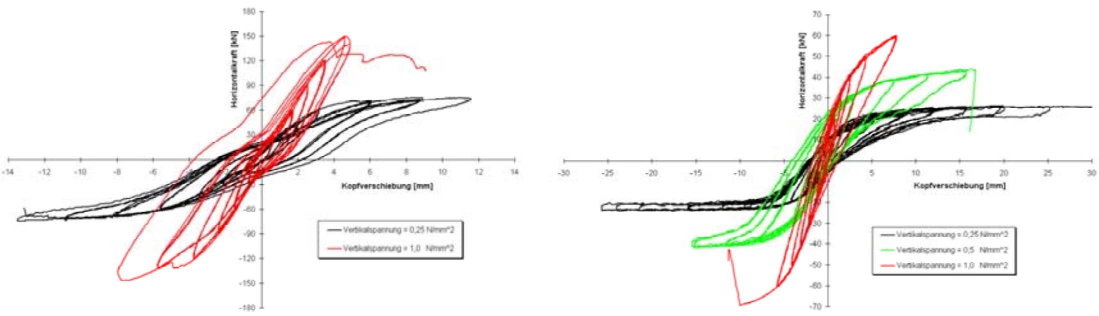

Figure 9. Cyclic load displacement curves of clay brick walls with a length of 1.10 m (left hand) and 2.20 m (right hand)

With these load displacement curves the aforementioned matrix depending on the vertical load level and the geometry ratio is build up within the first database as depicted in Figure 10 on the left hand side. Because of economic reasons just five experimental results with different length to height ratios and different vertical loads are available. Therefore the practical application is restricted to suitable ground plans with individual shear walls whose capacity can be determined by means of interpolation.

For this reason and first of all for the purpose of comparison a second database is generated containing capacity curves derived from analytical approaches described in section 3.1. The second matrix comprises capacity curves with additional vertical load levels and the geometry ratios as depicted in Figure 10 on the right hand side.

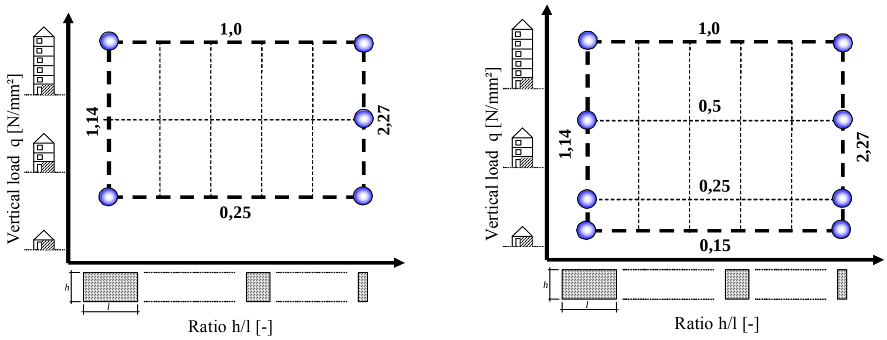

Figure 10. Matrix of experimental (left hand) and analytic determined capacity curves (right hand)

The analytical capacity curves are idealized according to FEMA guideline 306 [9] with respect to different failure modes. Table 3 shows load bearing capacity and maximum displacement of the experimental investigation in comparison with the analytical approaches exemplarily for two wall configurations. The material parameters and the experimental results of the walls with a length of 1.10 m and vertical loading of 0.25 N/mm² (W 25) and 0.5 N/mm² (W 50) are assumed according to [6].

Wall	**Experimental**		**Analytical solution (FEMA guideline 306 [9])**						**Difference**	
	V_{max} [kN]	**s_{max} [mm]**	**V_r [kN]**	**V_{bjs1} [kN]**	**V_{dt} [kN]**	**V_{tc} [kN]**	**V_{max} [kN]**	**s_{max} [mm]**	**V_{max} [%]**	**s_{max} [%]**
W25	25	24,8	19	65	66	16	19	23	24	7
W50	42	15,2	38	85	77	28	38	23	10	50

Table 3: Load bearing capacity and ductility of the experimental results in comparison with the analytical approaches

The stiffness of resulting bilinear idealisation illustrated in Figure 11 is calculated according to analytical formulas under consideration of the shear deflection [3] taking the stiffness degradation due to damage not into account. However the analytical approach fit the curve progression very well. Due to the overestimated deformation capability the maximum drift ratio is for further investigation generally limited to a level of 0.002 h_s/l.

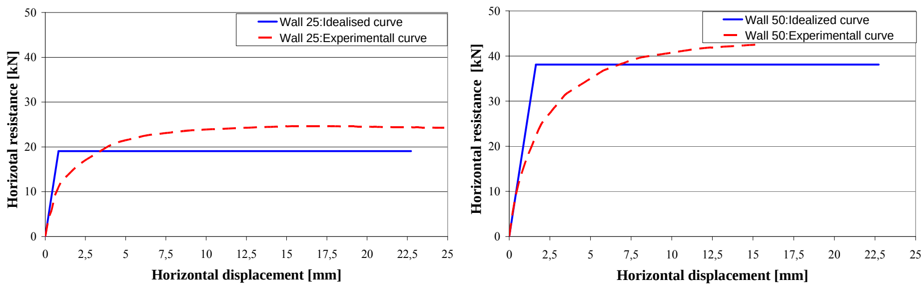

Figure 11. Capacity curves of experimental results in comparison with analytical approaches

5.2 Application example: Terraced house with three storeys

In the following the capacity spectrum method is applied to a typical terraced house with three stories and an inner storey height of 2.50 m. The dimensions of the house are 6.5 m x 11.0 m (Figure 12). The self weight of the reinforced concrete slabs is 5 kN/m² and furthermore a live load of 2.7 kN/m² is applied on each floor level. The seismic action in earthquake direction is represented by a linear response spectrum according to DIN 4149:2005 [4] for seismic zone 3 and subsoil condition A-R. The building capacity in main direction is affected by four walls at a length of 1.10 m (W2), and two walls at a length of 2.20 m.

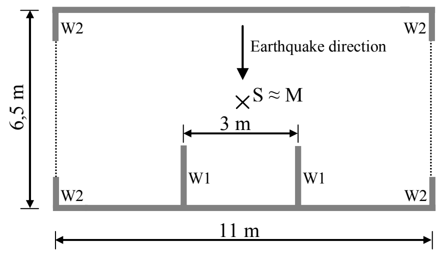

Figure 12. Ground plan of the terraced house

By means of interpolation the loading conditions of the single shear walls meet the requirements of both in section 5.1 introduced databases. The effective viscous damping of the experimental data implies the equivalent viscous damping due to hysteretic behaviour. In contrast, the idealized capacity curves solely take 5% viscous damping into account.

To draw a comparison the capacity curve and the material damping of the building is derived from both developed databases (see Figure 10). By transforming the response spectrum and the capacity curve into the S_a-S_d-diagram the "Performance Point" as intersection point of both curves was calculated iteratively (Figure 13).

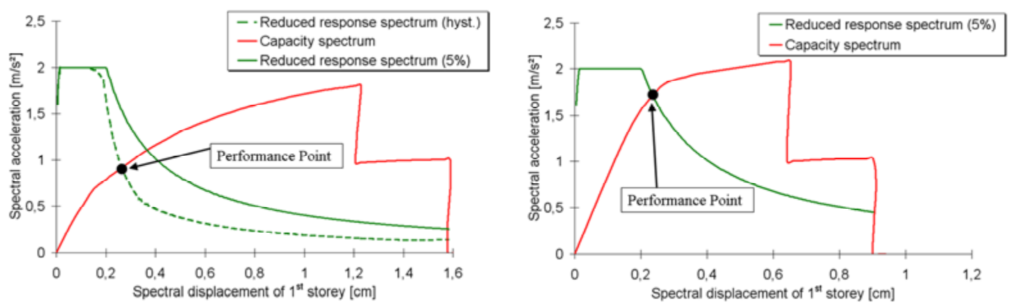

Figure 13. "Performance Point" of the terraced house related to experimental data (left hand) and idealized capacity curves (right hand)

In both cases the “Performance Point” lies in the ascending range of the capacity spectrum. The shape of the capacity curve in the diagramm on the left side indicates gradual stiffness degradation. By contrast the bilinear approach of the analytical approaches leads to the linear part of the capacity spectrum and higher spectral acceleration with lower spectral displacements. The staircase-like shape of the capacity spectrum reflects the successive failure of the walls. The first step corresponds to the failure of the walls W1, the following step relates to the failure of the walls W2. Nevertheless the structural stability is ensured for the given seismic action.

Hence sufficient structural stability is assured even if idealised capacity curves with reduced ductility and material damping are used. Actually the building exhibit further nonlinear bearing reserves taking the hysteretic damping and the real ductility into account.

6. CONCLUSIONS

The present paper introduces a new displacement based design approach for masonry structures based on the capacity spectrum method taking real existing load-carrying capacity and deformation capacity of masonry into account. The building capacity curve is calculated iteratively using capacity curves of single shear walls. The wall capacity curves are stored in a well structured database depending on material, geometry and loading conditions. Due to the large number of combinations the curves have to be obtained by cyclic shear wall tests in combination with nonlinear numerical simulations and analytical solutions. Therefore feasible analytical approaches are introduced and successful used for determination of idealised capacity curves for one material combination. The introduced displacement based design concept is implemented into the user friendly software package M-DESIGN. This facilitates structural engineers to perform seismic safety verification of masonry buildings considering the nonlinear bearing reserves.

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